A FUZZY SETS APPROACH TO INVENTORY SYSTEMS DESIGN

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CERTIFICATE

This is to certify that this work entitled 'A Fuzzy Sets Approach to Inventory Systems Design' has been carried out under my supervision and that it has not been submitted elsewhere for the award of a degree.

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Rajesh Dube

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SYNOPSIS

The present work deals with the development of an integrated inventory classification, forecasting and control system. This work is the first attempt to cover the gap between the concepts of Selective Inventory Management and the way it is practiced in the form of annual rupee usage based ABC analysis. Also, it is for the first time that the theory of 'Fuzzy Sets' has been used for inventory systems design. Using this theory a computerised inventory classification system has been developed. The proposed system is quite compatible with the concepts of Selective Inventory Management which emphasise the allocation of management control effort according to the overall relative importance of the items and not simply on the basis of annual rupee usage as is done in the classical ABC classification.

The overall importance of various items has been represented by feature vectors and the 'Fuzzy ISODATA' algorithm proposed by Bezdek and Dunn has been used for classification. A hospital medicine inventory situation has been studied and three groups of medicines with varying degree of importance have been found. The first group consists of those medicines which are important

from all considerations, the second group has the medicines which have low annual rupee usage and high expiry time but are otherwise important, and the third group comprises of the medicines which are unimportant from all considerations. The proposed classification scheme has been found superior to the traditional ABC analysis.

An adaptive control limit system has been designed for the first two classes of important medicines. A forecasting technique given by Buchan and Koenigsberg and a modified version of De Matteis's Part Period Algorithm have been used for lot sizing. An extension of Tchebycheff's inequality has been used to set reorder points for specified service levels. A more economical static control limit system has been proposed for the third group of unimportant medicines. A computer program has been developed for all the three classes of medicines.

This approach can be easily extended to cover other pure inventory situations like warehouses, grocery shops, super markets, etc. and should result into significant improvements in the existing cost structures and service levels.

CHAPTER I

INTRODUCTION

Economic development provides increased output and efficiency through specialisation. In the caveman's society, each man produced all of his own goods - food, clothing and a few utensils. In modern industrialised societies, each individual produces only a minute fraction of the goods and services he requires. Due to the spatial and temporal separation created by this high degree of specialisation, such a society cannot operate without large stocks of inventories. Inventories provide the buffer which allows these separations to exist. They perform the basic function of decoupling the operations involved in converting inputs into outputs.

A price has to be paid for everything one gets, and inventories are no exception. Inventories tie up assets and additional direct charges accumulate because of the storage space requirements, insurance, taxes and so on. Firms, therefore, cannot afford to keep unrestricted inventories at every point in the production process despite the benefits they bring. Even with closely controlled inventories,

manufacturing firms often find that as much as 25 to 50 percent of their total assets are invested in inventories. Moreover, wholesalers and retailers occasionally find as much as 75 to 80 percent of their assets in inventories. A survey of 1000 Joint Stock Companies by the Reserve Bank of India revealed that more than Rs. 1000 crores is blocked in inventories alone.

It cannot be overemphasised that decisions regarding the inventory are very important. Inventory models and mathematical description of inventory systems provide a basis for these decisions. Scientific inventory control is concerned with using mathematical models to obtain rules for operating inventory systems so as to minimise the sum of the various costs involved.

The present work examines a special type of inventory situation. An attempt has been made to devise a practical, efficient and readily adaptable system for controlling medicine inventory in hospitals. The proposed system can be used with some modifications in situations like drug stores and warehouses, supermarkets and retail shops. It is important to note that in industry the criteria usually have economic origins but in hospitals it is necessary to consider life and death as well as economic criteria. Practice most often has been to assign economic costs to all factors. In this presentation,

the influences of various factors on maintaining of inventory of medicines is considered in a scientific way through the proposed model rather than an arbitrarily assigned cost.

Some of the important problems encountered in designing a medicine inventory control system are;large number of items, high and varied service levels, consideration for noneconomic factors and substitutability and perishability of items. The situation is further complicated by seasonality and trends in the demand pattern, special storing methods for some medicines and variability in lead time.

The first phase of the work deals with classification of medicines in various importance groups for deciding the level of inventory control effort by the management for various classes of medicines. The second phase consists of the selection of appropriate procedures for forecasting, lot sizing and reorder point setting for a (R, Q) type system, depending upon the desired service levels and lead time variability.

1.1 The Item Classification Scheme

With thousands of items in inventory, it is often not possible or even desirable to exercise close control over all the items. One method frequently used, to determine

to decide the amount of attention to be paid to an item on this basis. Moreover, the percentage of items to be put in a particular class is arbitrary. Highly dissimilar items from other considerations for control may be grouped together. This reflects upon the need to group items in a more general and systematic way. An attempt has been made in this work to develop a general approach for classification which takes into account all the pertinent factors contributing to the relative importance of the item.

The vague and subjective nature of the term 'importance' has been the prime source of motivation for using the recently developed theory of 'Fuzzy Sets' [1] in this work for classifying items in various importance groups. The fuzzy classification scheme takes into account the imprecision associated with real-life systems. The central idea of this theory is the assigning of a membership grade between 0 and 1 to every member of a fuzzy set. Each item is represented by a feature vector whose components consist of various factors to which the importance of the item can be attributed. 'Fuzzy ISODATA' clustering algorithm has been used to identify the intrinsic structure of the feature vector data. The outcome is the three distinct groups of items. First group consists of items important from all considerations, the second group is of items which have low

annual rupee usage but are otherwise important, and the third group consists of unimportant items. The results of this classification scheme have been compared with those of the traditional A B C classification scheme. The information obtained about the various groups present in the data has been used to develop a procedure for forecasting and lot sizing.

1.2 Forecasting the Requirements

A key aspect of any decision making situation is being able to predict the circumstances that surround that decision and that situation. Such predictions, generally handled under the title forecasting, have been identified as a key subpart of the decision-making process. A number of forecasting methods or techniques have been developed during the last two decades. Selection of proper method plays a vital role in the successful application of the knowledge in this field.

It is a common misconception that the overriding criterion in selecting a forecasting method for a particular situation should be the accuracy of the available methods. Several other aspects need to be examined before any method is finally selected. The characteristics of forecasting methods that have been found most helpful in making the

judgemental decisions are; time horizon for which the forecast is to be made, demand pattern, model type inherent in the method, accuracy, applicability, and associated costs in terms of computational effort, data collection and storage requirements.

The hospital medicine inventory management problem requires a method capable of making medium range time horizon forecasts for a demand pattern with seasonality and trend. The method must be computationally efficient as demands for a large number of items are to be regularly forecasted. The present work uses a simple method proposed by Buchan and Koenigsberg [2]. The monthly forecasts thus obtained have been used as an input for determining lot sizes for the various items.

1.3 Lot-Sizing

A large number of mathematical models have been developed during the past years to study various inventory situations but not all of them are of practical significance. Besides the classical EOQ model and its variations which use differential calculus approach, simulation and dynamic programming have also been employed to solve complex inventory control problems. Simulation requires the formulation of an inventory model as a quantitative measure of performance and objectives. The relationships existing within and outside the firm are

quantified and optimal results are obtained by changing those variables over which control is possible until the best solution is found. Dynamic programming can provide better results than the static EOQ model when the demand is unstable. However, Dynamic Programming has its own limitations in terms of computer time and memory requirements and it does not appear to be a good approach when the number of items is large. In this work, a modified version of the part periods algorithm [3] has been used. This algorithm takes care of varying demand and is also computationally efficient. However, for unimportant items, the classical EOQ formula has been used.

1.4 Reorder Levels

The reorder levels are established to cater to the demand during lead time. The procedure used in this work takes care of the forecast error and variation in lead time. Tchebycheff's inequality has been used to determine the reorder points for desired confidence levels. To be economical in exercising the control effort, different procedures have been suggested to estimate lead time and its variance for different classes of items.

1.5 Organisation of the Thesis

After this brief introduction we shall review in Chapter II the literature relevant to the present work. Chapter III is devoted to describing the models contained in the present work and the resulting inventory control system. In Chapter IV, the applicability of the approach developed in this thesis is demonstrated through a case study from the Health Centre, Indian Institute of Technology, Kanpur.

CHAPTER II

LITERATURE REVIEW

The literature relevant to the present work encompasses ABC inventory analysis, the theory of fuzzy sets, cluster analysis, fuzzy clustering, forecasting, inventory control and the design of computerised inventory control systems. In this chapter we shall briefly review the important developments in these areas.

2.1 A B C Analysis

A B C analysis is a basic industrial engineering tool. It involves ranking the items of a population in descending order of some exhibited activity and then developing the most appropriate technique of handling the high-activity 'A' group of items, with perhaps different techniques for handling the medium activity 'B' items and the low activity 'C' items. In many cases, such differentiation in techniques for physical handling, control, or management gives substantially superior results to treating both the important and the unimportant items in the same way.

A B C inventory classification was first introduced by H. Ford Dickie [4,5,6] of General Electric Company, USA in 1951. It is an adaptation of Pareto's law.

Vilfredo Pareto (1842-1923), an Italian economist and sociologist, in a study of the distribution of wealth and income in Italy in 1897, observed that a very large percentage of the total national income was concentrated in a small percentage of population. Believing that this reflects a universal principle, he formulated the axiom that the significant items in a group normally constitute a small portion of the total items in the group and that the majority of the items are of minor significance. This law has been observed to be particularly true in inventory situations.

In many inventory situations approximately 20 percent of the items account for 80 percent of the total cost (unit cost times the usage quantity). In the typical ABC classification, these are designated as A-items, and the remaining 80 percent of the items become B's and C's, representing the middle 30 percent that account for 15 percent of the cost, and the bottom 50 percent that account for 5 percent of the cost respectively. However, there is no fixed convention as to what constitutes the A, B or C category. The proportions may vary from situation to situation. A recent paper by Herron [7] discusses some industrial engineering applications of the ABC-curves drawn from such analysis.

There have been hardly any improvements in the rupee usage based ABC classification technique since its inception. Though the concept emphasises taking into account all pertinent factors contributing to the importance of items and not to base the analysis only on rupee value, due to lack of mathematical tools available for dealing with fuzzy notions embedded in terms like 'importance', perhaps no improvement could take place. We have for the first time in this work tried to develop a generalised approach to the ABC type classification problems. Our approach is based on the recently developed theory of fuzzy sets and it uses fuzzy clustering algorithms for classification.

2.2 Fuzzy Sets

The concept of fuzzy set, first introduced by Zadeh [1] in 1965, is a generalisation of the concept of a conventional set. While in conventional set theory, objects are allowed only either to belong or not to belong to a set, in fuzzy set theory they are allowed to have a 'degree' of belongingness ranging from non-belongingness to belongingness to a set. A fuzzy set is characterised by a membership function which assigns to each object a grade of membership ranging between zero and one with respect to the set. The notion of fuzzy set provides a natural way of dealing with problems in which the absence of sharply defined criterion

of class membership is the source of imprecision which makes them immune to conventional non-fuzzy techniques of solution. Fuzzy sets, according to Zadeh [8], shall prove to have valuable applications in management science, economics, information retrieval, artificial intelligence, medicine, biology, linguistics and psychology.

The past decade has seen rapid developments in the theory of fuzzy sets. A fairly exhaustive introduction to the theory and applications of fuzzy sets [9] lists 238 papers dealing with a great variety of recent investigations. Bellman and Zadeh [10] have illustrated the applicability of fuzzy sets in decision making with fuzzy goals and constraints. Capocelli and DeLuca [11] have examined fuzzy sets in the context of decision theory. Goguen [12] was the first to introduce 'L-Fuzzy Sets' and the concept of 'entropy' of fuzzy sets was first defined by De Luca and Termini [13]. In reference [14] De Luca and Termini extended the concept of entropy to L-fuzzy sets. Another paper by De Luca and Termini [15] discusses many algebric properties of fuzzy sets. Rosenfield [16] and Yeh et al. [17] deal with fuzzy graphs in clustering. Other useful references on the subject include books by Kaufmann [18] and Zadeh et al. [9] and papers by Zadeh [19, 20], Bezdek [21,22,23], De Luca and Termini [24] and Capocelli and De Luca [25].

2.3 Cluster Analysis

There have been several applications of fuzzy sets in devising clustering algorithms. The primary objective of clustering techniques is to partition a given set of data into homogeneous clusters such that all points in the same group are similar to each other and dissimilar to those in the other groups. With the emergence of fuzzy algorithms, now all clustering techniques can be divided into two main classes, namely, hard (i.e. conventional) clustering methods and fuzzy clustering methods. Hard clustering techniques assign each object to one and only one of the clusters, but fuzzy clustering techniques assign a membership grade between 0 and 1 in every cluster to each object. Higher the membership grade of an object in a fuzzy cluster, stronger is its claim for belonging to that cluster.

2.3.1 Hard Clustering Techniques

The partitions generated by clustering, irrespective of whether they are 'hard' or 'soft', can be used in two ways. Either the partitions may be used to investigate the existence of families as done in numerical taxonomy, or they may be used as categories for classifying future data points as done in pattern classification.

Some of the important contributions dealing with taxonomical applications of clustering are by Sokal and Sneath [26], Jardine and Sibson [27], and Clifford and Stephenson [28]. A book by Duda and Hart [29] gives a detailed account of clustering techniques relevant to pattern classification. However, there are not many books dealing exclusively with cluster analysis. Two of these, by Anderberg [30] and Hartigan [31], are application oriented and give a detailed account of computer based clustering techniques. An extensive bibliography has been provided by Duran and Odell [32]. A paper by Kennedy [33] deals with applications of cluster analysis in industrial engineering.

2.3.2 Fuzzy Clustering Techniques

Fuzzy clustering is a relatively new area. The motivation for adopting fuzziness in clustering comes from the fact that in real life, the data is rarely so distinct that every member of the data set could be described as a full member of a single class. A fuzzy model enables us to overcome this objection by allowing every individual to have partial membership in all the clusters.

Clustering with fuzzy sets was first proposed by Bellman, Kalaba and Zadeh [34]. The papers by Chang [35],

Wee [36] and Gitman and Levine [37] discuss some of the earliest fuzzy pattern classification schemes. In 1969 Ruspini [38] delineated the first fuzzy clustering method based on objective functions, and foreshadowed the usefulness of information measures (entropy) in the context of fuzzy sets. He further enlarged and illustrated his technique in references [39, 40, 41]. In reference [42] Dunn has defined the first fuzzy extension of the classical within group sum of squared errors (WGSS) objective functional. This algorithm is now known as 'Fuzzy ISODATA' process in literature after the hard ISODATA process of Ball and Hall [43] which is a special case of the former. Bezdek [44] has generalised this approach to yield an infinite family of fuzzy algorithms. Dunn [45, 46] has given further results on fuzzy ISODATA. In reference [47] Bezdek has given an interesting physical interpretation of fuzzy ISODATA process. Methods of clustering based on fuzzy graphs are still in their infancy, references [16,17, 19,48,49,50] are seminal works in this direction. Some of the important applications of fuzzy clustering include plant classification [51], medical diagnosis [52], and design retrieval, group layout and evolution of composite components [53].

2.4 Forecasting

A large number of forecasting techniques have been developed during the last decades to fit the varied situations. Wheelwright[54] has suggested that these techniques can be grouped into two broad classes: quantitative techniques and qualitative or technological forecasting techniques. This classification generally reflects the extent to which a forecast can be based directly on historical data. Quantitative techniques are based on usage of past data whereas qualitative techniques are used when past data is not available.

2.4.1 Quantitative Forecasting Methods

In the area of quantitative forecasting methods we find a number of techniques whose common element is that the forecasts are based almost exclusively on past data. Some of the more widely used techniques in this class include moving averages, exponential smoothing, adaptive forecasting and regression analysis. Wheelwright and Makridakis [54], Trux [55], Box and Jenkins [56] and Brown [57] have discussed these in detail.

The simplest of all the forecasting methods are the moving average techniques. The exponential smoothing techniques are basically moving average techniques but they

represent all of the data and therefore are statistically stable. Exponential smoothing gives weightages to the data proportional to their age in an exponential fashion. Winter's three parameter exponential smoothing model [58] and Brown's single and multiple smoothing models [57] are significant contributions in this area. These models cover regular demand with stable trend and seasonality. Other variations of these techniques take into account the irregularities in demand and the nature of the market. Forecasting of sporadic demand, strongly fluctuating demand, sporadic demand with homogeneous and heterogeneous markets and regular demand with heterogeneous markets are included in such variations. Trux [55] has described these cases in detail.

The ability of an exponential smoothing model to track changes in the time series is dependent on the value of the smoothing constant. A relatively large value of the smoothing constant (close to unity) will cause the forecaster to respond quickly to changes in actual demand, reflecting a fraction of random changes in demand as well as actual shifts in the average demand. A small value of the constant (close to zero) will respond more slowly and smoothly. Therefore a fixed value of the smoothing constant in case of varying demand is likely to give erroneous demand estimates. Adaptive models overcome this objection to a great extent by

continually revising the value of the smoothing constant when changes in time series occur. Chow [59], Dudman [60], Trigg and Leach [61] and Roberts and Reed [62] have contributed in this direction. Jain [63] has developed a hybrid forecaster using simple exponential smoothing and tracking signal model of Trigg and Leach. This hybrid forecaster enjoys the strong points of both the forecasters as they are mutually compensating. The latest addition to the family of adaptive forecasters is the adaptive filtering model of Wheelwright and Makridakis [54] which assigns weightages to past data such that the mean squared forecast error is minimum. Rao and Shapiro [64] have developed a highly sophisticated adaptive forecaster using evolutionary spectra.

Another class of sophisticated quantitative techniques is causal models. In these models the relevant causal relationships are expressed mathematically. Leading indicators and regression models [57] and life cycle analysis [65] are the commonly used causal models. Some other complex quantitative models are Box and Jenkins method [56] econometric forecasting and input-output tables [54].

2.4.2 Qualitative Forecasting Methods

Qualitative forecasting techniques are used when no historical data is available. The aim of these methods is to

forecast changes in a basic pattern as well as the pattern itself. They are used primarily in two types of situations. First, to forecast when a given new process or product will become widely accepted and secondly for forecasting the new developments and discoveries that will be made in a specific area. Because of the difficulty and cost of working with qualitative methods of forecasting, they are generally applied only to long-term situations and to those of major importance to the firm.

The mechanics of these techniques consists of interpretation of the related data by 'experts' who are supposed to have the best understanding of the situation and ability to predict changes of a similar nature. Market research [66], historical analogy [66], Logistics and S-curve approach [67], time independent technological comparison [68], morphological search [69], Delphi technique [70, 71] and the relevance tree method [72] are some of the prominent qualitative forecasting methods.

2.5 Inventory Control Techniques

Inventory control has been one of the most favourite areas of interest for research workers and more analytical research has been directed towards inventory control than any other problem area in business and industry. Though the first

attempt dates back to 1912, significant progress has been made only after 1950 and very many realistic approaches to inventory control theory have been proposed. The inventory control problem has been tackled by analytical, graphical and simulation techniques and a good amount of research has been done in these fields. Recent application oriented research has been concerned over devising computationally efficient 'dynamic lot-sizing' algorithms for realistic situations in which the demand, rate of return and prices are variable.

2.5.1 Analytical Techniques

Newberry [73] has given a thorough account of the early developments in inventory control theory. In 1912 Babcok for the first time tried to determine a mathematical basis for economic lot size by using a cubic equation. In 1915, Harris developed an economic lot size formula which is almost identical to its present form. In 1918 Taft modified the economic lot size relationship to account for the fact that manufacturing and sales period overlap each other since some of the production items are diverted to meet current orders without entering the inventory. Very few new developments were recorded between 1920 and 1931 with regard to the economic lot size formula. In 1931 Raymond suggested an approach to account for all the factors that

affect economic lot size and gave limits of possible lot sizes with little difference in unit costs. Around 1938, Scherider, Erich and Jessen tried to further generalise the problem by considering manufacturer's sales forecast, production capacity constraints, initial inventory, carrying charges and production costs.

There were few, if any, major developments in the field of inventory control between 1938 and 1951. The attempts prior to 1951 considered deterministic problems but after 1951 researchers tried more realistic approaches by incorporating stochastic nature of demand in their models. Arrow et al. [74] in 1951 and Dvoretzky et al. [75,76,77] in 1952 and 1953 presented pioneering work in this direction. Bellman et al. [78] in 1955 analysed the dynamic inventory situation with linear ordering and penalty costs. In the subsequent developments the stochastic nature of lead time was also incorporated which made it necessary to consider the determination of reorder point together with order quantity. Since the reorder level requires stock to be reviewed to trigger a new replenishment order, and this can be done either continuously or at fixed interval of time, the further developments branched off based on the review procedure.

The essence of continuous review models is to monitor the stock continuously as the demand occurs. This involves the determination of a reorder level, s, and the order quantity, Q, giving rise to two systems; the (s, S) and (s, Q) systems. In the (s, S) system an order, Q = S - s, is placed when the stock depletes to reorder level. Inventory, therefore, fluctuates between two levels 'S' and 's'. In the (s, Q) system, a fixed quantity 'Q' is ordered when the inventory falls below the lower level 's'.

The periodic review models are similar to continuous review models except that instead of monitoring the inventory status continuously, the review is carried out only after a time period 'T'. When 'T' is very small, the periodic review system tends to become identical to the continuous review system.

A hybrid system, which is a combination of the above mentioned systems, also exists. This system is known as (s, S, T) or (s, Q, T) system. In this system, the replenishment is provided at every period 'T', but if in between the review periods the stock declines to 's', an order is placed to replenish the stock.

Further literature is presented in the light of the above stated policies.

Periodic Review Models

The fixed cycle policy has received very little attention from the researchers because, theoretically, if the review costs are ignored, all other things being equal, a continuous review system always has a lower average annual cost than a periodic review system. However, this system still exists because it is not always feasible to make continuous reviews for low cost high volume products.

Vazsonyi [79] has given an excellent analytical discussion for determining the maximum level, S, when the length of review period is given and demand is variable. Whitin [80] determined the optimal length of review period when the demand is variable. He calculated the upper level, S, by adding the expected demand during the length of the period to the safety stock required for any desired level of protection. Naddler [81] and Churchman [82] have presented some simple models with variable demand which determine optimal value for the maximum level, S. Iglehart [83] investigated both back-order and lost sales cases and incorporated a variety of demand distributions. Yaspan [84] developed a model where the demand is a function of both the time required to fill the order and the nominal value. This is a realistic assumption because customer is less likely to order in future if he has to wait for back orders to be satisfied.

Continuous Review Models

Continuous review models have received a very wide attention. Arrow, Harris and Marschak are the pioneers in this field. In their classical work [74], they considered fixed models, uncertain models, static models, dynamic models and also derived a procedure for obtaining optimal values of 's' and 'S' in the (s, S) policy when the length of period is specified. Dvoretzky, Kiefer and Wolfowitz [75, 76,77] have proposed three highly theoretical inventory control models. The first model is concerned with determining the optimal ordering policy when the demand distribution is known; the second model deals with the optimal ordering policy when demand distribution is unknown, and the third model establishes the necessary and sufficient conditions for the validity of the (s, S) policy. All these models are highly mathematical and rather abstract and are not suitable for direct application purposes.

In the literature, a few models which account for the safety stock required for varying levels of protection have also been reported. Vazsonyi [79] determined the safety stock for normally distributed demand, while Whitin [80] tackled the problem for specified service level and Poisson demand. Buchan [2] calculated safety stock for specified service levels and for uniform, normal and

exponential demands. Llewellyn [85] gave a cost model to calculate the lot size to be produced in order to compensate for scrap losses in manufacturing.

The foregoing works give a general approach to inventory problems. In recent years, different models for specific problems have been developed. Lampkin [86] solved the problem for spare parts required for aircraft overhauling with Poisson demand and lead time with gamma distribution. A good number of models have been devoted to blood bank inventory control [87]. After the publication of perhaps the first paper by Klein and Rosenberg [88] on perishable inventories in 1960, it seems to be one of the live areas of current inventory research. Two recent papers in this direction are by Fries [89] and Nahmias [90].

In somewhat unconventional approaches, the inventory problems have been solved by queuing theory. Cohen [91] treated demand as an arrival process and replenishment as a service facility. On the contrary, Karush [92] treated demand as a service facility and replenishment as an arrival process. Decision criteria were server busy time, customer balking rate, waiting time for replenishment, etc.

At present, the research work on inventory problems is being carried out at many different levels. At one

extreme, a considerable amount of work is concerned strictly with practical applications, while at the other extreme, work is being done on the abstract mathematical properties of inventory models without any regard for possible practical applications. The practical situations demand simple and computationally efficient methods and the graphical and dynamic lot sizing techniques are oriented towards this.

2.5.2 Graphical Techniques

Often mathematically sophisticated models prove to be futile in the face of the crudeness of available data. This difficulty has motivated researchers to carry out application oriented work including graphical, semi-graphical and empirical methods. Among the notable works are those by Herron [93], Eckey et al. [94], Hadley and Whitin [95] and Buckland [96].

Herron, using graphical and algebric techniques, has attempted the problem with stochastic demand and lead time by considering the cases of stockout penalty proportional to the number of stockout occasions and to the annual number of units out of stock.

Eckey et al. give a semigraphical method for Poisson demand and lead time. Hadley and Whitin suggested an

iterative scheme for lost sales with stochastic lead times and demand. Buckland developed a nomograph for normal demand and lead time and specified service level.

2.5.3 Dynamic Models

Another class of practice oriented inventory models is dynamic lot sizing procedures. This type of procedure is called dynamic because it allows the time dependent pattern of orders and the rate of return on capital, as well as the pattern of the unit price dependent upon the quantity, to be taken directly into calculation as variables, whereas in the classical lot-size formula they are all constant. The basic concepts of the method are as follows.

In order to find the lot size for minimum total cost, we start from an order quantity whose total cost has been worked out. This order quantity is then systematically adjusted and changes in cost arising from the adjustments are assessed. We then proceed in the direction of falling cost until the desired minimum cost is found.

The optimum lot size is therefore discovered by means of systematic tests in contrast to the classical lot-size calculation, which establishes the minimum cost by analytical methods which are feasible only in limited circumstances.

The Part Period Algorithm (PPA) developed by De Matteis [3] in 1968, is the first effort in this direction. Its performance compares well with that of the Wagner - Whitin algorithm [97] which is based on dynamic programming. Though the Wagner-Whitin algorithm gives the best results and is used as a standard to compare the performance of various dynamic lot sizing procedures, it entails high computational burden and is too complicated to be understood by the average system user. The Part Period Algorithm overcomes these drawbacks to a great extent and the inventory cost determined by it is only very slightly more than that from the Wagner-Whitin algorithm.

However, PPA is also not free from objections. Orlicky [98], who has presented an excellent review of various lot sizing techniques, has indicated some basic flaws in PPA. The PPA assumes that inventory depletion occurs at the beginning of each period, and a portion of each order equal to the first period requirement is consumed immediately upon arrival in stock and thus incurs no inventory carrying charge. The inventory carrying charge is thus computed on the basis of this assumption rather than on average inventories in each period. Orlicky has further criticised the 'look ahead' and 'look back' correction routines of PPA and has shown with examples that these special features

of PPA do not improve its effectiveness but on the conterary are dubious.

Another work in this direction is by Pierce [99]. He considered a multi-item economic lot-sizing problem in which the ordering policies for individual items are interdependent as a consequence of joint ordering or production set up cost. The problem is to determine an optimal ordering plan in which the sum of the costs of carrying inventories and the costs of ordering are minimised and in which the known demands for each item in each time period are satisfied. He has presented two algorithms to solve this problem. The first one is a direct algorithm which yields periodic solutions and applies to problems in which demand occurs uniformly over time. The second algorithm is based on dynamic programming and is an extension of Wagner-Whitin algorithm.

Trux [55] has also discussed several dynamic lot sizing techniques which are very useful from the view point of computerised inventory control systems design.

2.5.4 Simulation Models

The analytical methods, generally, are very complicated if the demand and lead time both are stochastic in nature. The simulation of inventory situations on digital

computers has become popular due to great speed of modern computers and sometimes it is the only suitable method to solve complicated inventory problems.

Quite a few developments have been reported in the literature regarding simulation of inventory situations. McMillan [100] gave a Monte-Carlo simulation study of a stochastic inventory problem. Cohen [91] formulated a multilocation model for a central store with branch warehouses. Gross and Soriano [101] carried out a simulation study comparing the resulting inventory reduction obtained when sealift is replaced by airlift. Lewis [102] simulated on a small hybrid computer industrial inventory situations in which demand and lead time are both stochastic.

2.6 Computerised Inventory Systems

In previous sections a number of inventory control techniques were discussed which reduce the managerial decisions at item level. This, however, expands the information for a single item to such an extent that it becomes almost impossible to carryout computations manually with speed and store the necessary variables and parameters for thousands of items in a perpetual inventory system. Also, time is seldom available to forecast demand and to review the stock status for each item. Therefore, the help of computer

is sought to handle the enormous amount of data at high speed. In past many attempts have been made to computerise a part or whole of the inventory system. Some of them are presented here in brief.

Blondin [103] computerised an A B C system of inventory control in 1964 using IBM 7090 and IBM 357 data transmission equipment. The A and B items were tightly controlled and the key problem was receiving upto the minute records of stock balances, particularly for A and B items.

A similar work has been published by Geschelin [104] regarding computerised inventory control used for Chrysler car assembly group comprising of seven assembly plants. A third generation IBM computer was used. The method of operation was based on Chrysler's concept of management by 'exceptions' such as 'shortages' and 'overshipments'. The exceptions were considered to be the cause of delay and, therefore, required actions.

Next work in computerised stock keeping is an IBM Hospital Inventory Program [105] where a perpetual inventory system provides for a continuous record of supplies, receipts and stock on hand. Items requiring management attention are listed on exception reports. On a periodic basis transactions posted to the inventory record are

transferred to general ledger account which reflects the aggregate of receipts and issues posted to individual inventory records.

In the foregoing works the computer effort was limited only to stock keeping and obtaining some useful reports. Further attempts have been made in the direction of greater automation to cover various management policies based on scientific inventory control techniques. One such effort has been in handling problems by LIMIT-Technique [106]. The LIMIT program makes it possible to obtain the most economical lot sizes for all items in a selected group of items where there is a limitation on the number of orders to be handled.

Reed and Stanley [107] have designed a procedure for controlling general hospital inventories. They have used ABC inventory control in which demands are forecasted for A and B items by simple exponential smoothing. They have, however, not tried to find lot sizes optimally but the orders equal to the next period demand forecast are placed in each period. They have given some indications on computerising the procedure.

Buchan [2] in his case study named 'Warehouse Inventory Management for a Drug Wholesaler', has given the objectives of the system design and outlined the procedure based on Selective Inventory Management Concepts (SIM). The SIM includes the consistency in the ordering of inventory items by the routine use of mathematical inventory formulae and selective allocation of available control effort to each item according to its importance.

In 1969, Barret [108] discussed automatic inventory control techniques which include forecasting system, calculation of order quantities for given discounts, allocating order quantities in a group order, setting safety stocks, etc. He also discussed the advantages and disadvantages of various control systems in the light of computer application.

The effort of Trux is of immense practical importance in the context of computerised purchasing and stock control. In his seminar papers, which were later published as a book [55], presented at the German Institute of Industrial Management, Frankfurt, he elucidated various algorithms, procedural organisations and programs details regarding data processing for purchasing and stock control and materials management information system.

Next important work in this area is of Fetter [109] who has given an integrated computer oriented inventory management system. This work presents a centrally managed multi-item inventory system using either continuous or periodic review systems. He has listed out the initial inputs and computer operations which include stock-reviews and calculation of reorder level and order quantity.

In a recent book Orlicky [98] has discussed the design and implementational aspects of computerised material requirements planning systems. Recently Puri [110] has developed a computerised ordering system for drug distribution business.

2.7 Summary

The review of forecasting and inventory control theory has shown that these problems can be solved by using a variety of approaches. The forecasting problem alone can be tackled by a number of techniques with varying degree of sophistication. The systems designer is further stranded by the vast number of models available in inventory theory. However, despite the wide spread use of selective inventory management concept, no effort has so far been made at evolving a general strategy which can classify items in terms of their relative importance taking all relevant factors into consideration.

Though a vast amount of research has been carried out in inventory control theory, published papers rarely devote enough attention to the need for a link between forecasting and reorder policy. Further, we find very few publications devoted to hospital medicine inventory control problem which is quite different from the general inventory control problems.

In view of the above, we in this work have tried to introduce a general approach to selective inventory management using fuzzy sets and an integrated forecasting and and inventory control system has been designed to meet the requirements of medicine inventory control situation in hospitals. In the next chapter the details regarding the approach used in this work will be described.

CHAPTER III

PROBLEM STATEMENT AND METHODOLOGY

The introduction and literature review have already given an overview of the problem. In this chapter the problem and the solution methodology will be discussed in detail. The chapter comprises of three sections dealing with the item classification scheme, forecasting procedure and ordering policy for medicine inventory. It is perhaps for the first time that a classification scheme based on the recently developed theory of 'Fuzzy Sets' is being used in Inventory Analysis. Being a new approach, a major portion of this chapter has been devoted to describe the classification scheme. It is felt that the forecasting and lot-sizing procedures are relatively familiar and therefore they are not dealt with in great detail.

3.1 Statement of the Problem

A general statement of the problem will be: 'Design a practical, efficient and readily adaptable procedure for managing medicine inventory in a medium-sized community hospital'.

The first logical step towards designing any system is to specify its objectives and requirements. Before this is taken up, it would be worthwhile to consider what makes a system effective and efficient. Is the success attributable to the skill exercised in design, or to the caliber of management during the operation stage? Successful systems are often characterised by their simplicity, flexibility, reliability, economy, and acceptability. While simplicity, flexibility and reliability tend to be function of design, economy and acceptability pertain to both design and operation. Though it may not be always possible to achieve this, the designer should aim at striking an optimal balance between these factors to meet the inventory management objectives.

3.2 Inventory Management Objectives

The objective of any inventory management system is to provide the required goods, in the right quantity, at the right place and at the right time, at minimum overall cost. Further consideration will show that this essentially boils down to answering the following questions:

(1) What should be ordered?

(2) When should it be ordered?

(3) How much of it should be ordered?

All inventory situations are alike so far as the above questions are concerned. However, each inventory problem will differ somewhat in the specific use of quantitative methods and the level of sophistication required in employing decision models. The supporting facts and forecasts will depend upon the unique characteristics of the situation under consideration.

The solutions are likely to be situation and problem oriented and this work is no exception. Though the basic ideas and methodology described here can be applied to many inventory situations, what follows has a bias towards the problems of the situation studied in this work. Having seen what is expected from an inventory management system in general, we may now proceed to examine the hospital medicine inventory situation.

3.3 The Hospital Medicine Inventory Control Problem

To be successful, a management control system has to be compatible with the specific problem situation. The important characteristics of the hospital medicine inventory anagement problem may be summarised as follows:

i) There is a wide variety of medicines.

ii) For some ailments, if the required medicine is not available, substitution by suitable medicines from stock is permissible.

(iii) In general, the requirements for medicines are seasonal with a definite trend.

(iv) Some of the medicines are frequently in short supply in market.

(v) The medicines have varying degrees of consequential importance in terms of probable loss due to non-availability in emergency.

(vi) A small percentage of medicines tie up most of the capital.

(vii) High service levels are recommended by hospital administration as nonavailability of a medicine may be the question of life and death for a patient.

(viii) The procurement lead time is variable for most medicines.

(ix) Due to government control, the prices can be considered stable for a reasonable planning period.

(x) Special storing methods are required for some of the medicines.

(xi) A good number of medicines, particularly injections, expire in due course of time.

(xii) The present hospital regulations make it imperative to maintain stock records for all medicines.

Though the above list may not be exhaustive, it nevertheless, gives a fair idea of the situation. The first

problem, as we have already seen, is how to deal with an inventory control situation when thousands of medicines are used. We have also to decide the level of mathematical sophistication to be employed in the inventory control system so that an optimal compromise is sought between the system operating cost and the savings due to it.

3.4 Selective Inventory Management Concept

Selective Inventory Management (SIM) deals with segregating inventory items into areas of relative importance. It, thereby, highlights the items for which the potential savings appear to be greatest and which should be controlled with utmost care. Only a careful and complete selectivity analysis in advance of any attempt at quantitative economic analysis will determine the extent to which the mathematical methods of inventory control should be applied. In such an analysis, each item in inventory should be ranked for importance in terms of pertinent characteristics. The procedure will vary from one inventory situation to another, it being necessary in some instances to consider all items to be of equal importance. An example would be a production or assembly line where service level is important to keep the line operating. Where selectivity is warranted, which is the usual case, classification of items on the basis of relative importance can be confirmed by an evaluation using an appropriate

mathematical model. The most commonly practiced, or perhaps the only known method for selective inventory management is the annual rupee usage based ABC analysis.

3.4.1 Traditional ABC Analysis

The basic idea behind the rupee usage based ABC item classification scheme is to apply the bulk of the planning and control resources to A-items, 'where the money is', at the expense of other classes which have demonstrably much less effect on the overall inventory investment. In a typical ABC classification the top 20 percent, middle 30 percent and the last 50 percent of the items may account for 80 percent, 15 percent and 5 percent of the total inventory investment respectively. However, there is nothing sacred about these figures and the proportions may significantly vary from situation to situation [7]. The steps required to perform an ABC analysis are:

(1) Calculate the annual usage in units for each item.

(2) Extend the annual usage in units by multiplying it with unit cost to get the annual usage in rupees for each item.

(3) Rank the items from highest to lowest annual rupee usage and assign categories.

This procedure has been computerised in the present work and the results have been compared with those obtained

from the proposed 'Fuzzy Inventory Classification Scheme (FICS)'.

3.4.2 The Need for a New Classification Methodology

It is unfortunate that during the past years this type of rupee usage based analysis has been indiscriminately applied with no regard for the needs of a particular situation. A good inventory classification system should recognise many other characteristics of the items in addition to their annual rupee usage. It would be absurd to conduct an ABC analysis and apply a rigid set of ordering rules based solely on this criterion. To cite an example, in a hospital medicine inventory control situation, judgemental decisions about the importance of an item only on this basis may lead to serious trouble. There are factors like life saving importance, substitutability, expiry time, storage requirements, market availability, lead time, demand and lead time variability and so on, in addition to annual rupee usage, which must be given due weightage in evaluating the relative importance of an item. As we have already seen, the principle of selective inventory management does not say that the judgement be based only on annual rupee usage. It simply seeks to apply maximum control effort to the vital few and allocate the control and planning resources in accordance with the relative importance of items. Further, the

assignment of a certain percentage of the items to any particular class is rather arbitrary. Even highly dissimilar items may be put together and subjected to the same control procedure even when they resemble each other only on one count, that is annual rupee usage. This calls for the need to evolve a methodology which takes into account all the pertinent characteristics of the situation, together with the inherent imprecision involved in the term 'relative importance', and scientifically classify items in various groups. To this end, a classification scheme based on the theory of 'Fuzzy Sets' has been developed during the course of this work.

3.5 The Fuzzy Sets Approach

More often than not, the classes of objects encountered in the real physical world donot have precisely defined criteria of membership. For example, the 'class of animals' clearly includes dogs, horses, cows etc. as its members and clearly excludes such objects as rocks, fluids, plants etc. However, such objects as starfish, bacteria, etc. have an ambiguous status with respect to the class of animals. The same kind of ambiguity arises in the case of the 'class of beautiful woman ' or the 'class of tall men'. Though they do not constitute classes or sets in the usual mathematical sense of these terms, the fact remains that such imprecisely

defined 'classes' play an important role in human thinking. The concept of 'fuzzy sets' is fast getting recognised as a powerful tool to solve problems involving such fuzzy ideas.

A similar situation is encountered in Selective Inventory Management where the items are to be classified on the basis of their relative importance. It is not easy to answer what should decide the relative importance of an item, what is meant by terms like 'most important', 'unimportant', and 'vital', and which items are 'important' and which are not. In some way the 'class of important items' is not any less vague than the 'class of beautiful women' or the 'class of tall men'.

The notion of a 'fuzzy set' provides a convenient point of departure for the construction of a conceptual frame work which parallels in many respects the frame work used in the case of ordinary sets, but is more general than the latter and may prove to have a much wider scope of applicability. Essentially, such a frame work provides a natural way of dealing with problems in which the source of imprecision is the absence of a sharply defined criterion of class membership. The theory of 'fuzzy sets' is on the verge of entering into an era of rapid development and several algorithms have been developed which have the potential to resolve the problems of 'fuzzy classification'.

One such problem is ABC type inventory classification which requires the formation of groups of items on the basis of their 'relative importance'. These types of problems come in the domain of cluster analysis which is a well known technique of grouping objects. The traditional cluster analysis produces disjoint clusters, that is, an object gets allotted to one and only one cluster. Such strict classification is inappropriate in most real life problems, like the one we are faced with, because of the imprecision or fuzziness present in the system. Fuzzy clusters account for this by assigning to every object in the cluster a membership grade between zero and one. The classification system developed in this work is based on the concept of fuzzy clusters and it successfully tackles the problem of forming item groups on the basis of relative importance.

We shall review now some important concepts in the theory of fuzzy sets and then describe how they have been eventually applied to solve our problem.

3.5.1 Fuzzy Sets and Fuzzy Clusters

The objective of cluster analysis is to group objects into clusters such that the elements within a cluster have a high degree of natural association among themselves while the clusters are relatively distinct from one another. With full faith in Pareto's law of 'vital few', we resort to

cluster analysis to study and reveal the various importance groups present in hospital medicine inventory. Our approach will be to first review some definitions, traditional cluster analysis, fuzzy clustering and its advantages, and then describe the fuzzy inventory classification scheme (FICS) developed in this work. The review is based on the paper by Bezdek [22].

Let, $X \subset R^s$, be a space of points (objects) with a generic element of X denoted by $\underline{x}$. Thus, $X = \{\underline{x}_1, \ldots, \underline{x}_n\}$. We shall call each $\underline{x}_i = (x_{i1}, \ldots, x_{is})$ a 'feature vector' whose j-th component, x_{ij}, represents j-th feature of the individual represented by $\underline{x}_i$.

Any 'fuzzy set (class)' A in X is characterised by a 'membership (characteristic) function $u_A(\underline{x})$' which assigns to each object in X a real number in the interval [0, 1]. The value of $u_A(\underline{x})$ at $\underline{x}$ represents the 'grade of membership' of $\underline{x}$ in A.

Conventional cluster analysis, also known as 'hard cluster analysis', deals with partitioning X into (c) non-empty subsets (clusters). The classification strategy is such that the partitioning is optimal with respect to some well defined classification criterion. Usually $2 \leq c \leq n$; where (n) is the number of feature vectors in X. Later we

shall see how the 'importance' of an item in inventory can be modelled into a feature vector and how the set of these 'importance vectors' can be partitioned through clustering algorithms in order to find various distinct but natural importance groups present in it.

The basic problem with all conventional clustering techniques is that even when working within the same metrical framework on X, they may yield different partitionings of X[42]. The inherent nonuniqueness of conventional clustering suggests opting for a fuzzy sets approach since fuzzy membership values incorporate a reflection of non-statistical uncertainty about the structure in the data [22]. Further, the fuzzy characteristic functions portray the mathematical inter-relationships of the individuals in X, better than the hard partitions.

Let V_{cn} be the vector space of all real (cxn) metrices with the following inner product, induced norm and norm metric defined in it.

$$\langle A, B \rangle = \text{trace}\,(AB^t) \tag{3.1a}$$

$$\langle A, A \rangle = \| A \|^2 \tag{3.1b}$$

$$d\,(A, B) = \| A - B \| \tag{3.1c}$$

$\forall$ A, B ε V_{cn}; the superscript (t) denotes the transpose operation. If, any U ε V_{cn}, satisfies the following conditions, we call it a 'hard c-partitioning of X'. Let u_{ki} denote the

ki-th element of U.

$$u_{ki} \in \{0, 1\} ; \forall\ 1 \leq k \leq c ; \quad 1 \leq i \leq n \qquad (3.2a)$$

$$\sum_{k=1}^{c} u_{ki} = 1 ; \forall\ 1 \leq i \leq n \qquad (3.2b)$$

$$\sum_{i=1}^{n} u_{ki} > 0 ; \forall\ 1 \leq k \leq c \qquad (3.2c)$$

The elements of the j-th row of U; $U_{(j)} = (u_{j1}, \ldots, u_{jn})$, are the values of any characteristic function on X, say $u_j: X \rightarrow \{0, 1\}$. The conditions, (3.2a) and (3.2b), together require disjoint intersections and collective exhaustion of X. Condition (3.2c) requires that each of the c-partitioning subsets is non-empty. The 'hard c-partitioning space, M_c, associated with X', which is also the basic model underlying most hard clustering methods, can be defined as:

$$M_c = \{ U \in V_{cn} : U \text{ satisfies conditions } (3.2) \} \quad (3.$$

As we have already pointed out the conventional clustering methods are handicapped by their inability to partition X uniquely because the resulting partitions are dependent upon the structure of the data as well as the clustering algorithm. Hence we shall resort to fuzzy clustering to overcome this predicament.

Moreover, in real life we are frequently faced with the problem of classifying objects which may have mixed attributes. It might be inappropriate to assign an object to a particular group as it may have a certain degree of belongingness to every group. Conventional clustering models fail to accurately represent situations involving such 'fuzzy uncertainty' in the structure of the data. However, the fuzzy model provides for such situations by allowing the individuals to have grades of membership in various groups. This can be achieved by simply extending the range of characteristic functions on X to the closed interval [0, 1]. Thus $u_k : X \rightarrow [0, 1]$ is a 'fuzzy subset (cluster)' in X and the values $u_{ki} = u_k(\underline{x}_i)$ are called the 'grades of membership' of the objects $\underline{x}_i$'s in a fuzzy cluster u_k. We call, $U \in V_{cn}$, a 'fuzzy c-partition of X', provided it satisfies the following conditions:

$$u_{ki} \in [0, 1] \; ; \forall \; 1 \leq k \leq c \; ; \quad 1 \leq i \leq n \qquad (3.4a)$$

$$\sum_{k=1}^{c} u_{ki} = 1 \quad ; \forall \; 1 \leq i \leq n \qquad (3.4b)$$

$$\sum_{i=1}^{n} u_{ki} > 0 \quad ; \forall \; 1 \leq k \leq c \qquad (3.4c)$$

The j-th row of U, denoted by $U_{(j)} = (u_{j1}, \ldots, u_{jn})$, gives the membership values corresponding to the fuzzy cluster

$u_j: X \rightarrow [0, 1]$. The 'space of the fuzzy c-partitions; M_{fc}, associated with X' can be defined as:

$$M_{fc} = \{ U \in V_{cn} : U \text{ satisfies conditions (3.4)} \} \quad (3.5)$$

It has been shown that $M_c \subset M_{fc}$. M_{fc} is, in fact, the convex hull of M_c in V_{cn} (Bezdek [44]) and is also compact. These properties have helped in the mathematical analysis of clustering schemes defined on M_{fc} [22].

By incorporating fuzziness, the individuals in X can now have full membership in a single cluster, or can claim partial membership in two or more fuzzy clusters. The condition (3.4a) allows the various clusters to have overlapping intersections. Condition (3.4b) requires each $\underline{x}_i$ to have full membership in X and (3.4c) requires that none of the fuzzy clusters is empty. The provision for an item to claim membership in several clusters simultaneously, is particularly helpful for the classification problems like the one we are dealing with. It enables us to figure out the natural pattern of 'importance distribution' in the feature vector data of the medicines and also tells us how strongly a particular medicine belongs to a particular 'importance group'. This latter information will be helpful in finally assigning an item to a particular importance group.

Now the question before us is, given M_{fc}, how can the fuzzy c-partitions of X be found? One nice way to optimally partition X is via the family of generalised WGSS (within group sum of squared) error objective functionals defined in [44]. On the cartesian product of M_{fc} with R^{cs} we define $J_m(U, \underline{V})$ for $m \in [1, \infty)$ as:

$$J_m(U, \underline{V}) = \sum_{i=1}^{n} \sum_{k=1}^{c} (u_{ki})^m \| \underline{x}_i - \underline{v}_k \|^2 \qquad (3.6)$$

where $U \in M_{fc}$; $\underline{V} (\underline{v}_1, \ldots, \underline{v}_c) \in R^{cs}$ and $\underline{v}_k = (v_{k1}, \ldots, v_{ks}) \in R^s$ for $1 \leq k \leq c$, and $\| \cdot \|$ is any norm on 'feature space'. The (c) vectors $\{\underline{v}_k\}$ comprising $\underline{V}$ are presumed to have features prototypical of vectors in X and have a high affinity for membership in the respective fuzzy clusters $\{u_k\}$. They are called 'centroids' or 'cluster centers' of their respective fuzzy clusters. The norm $\| \cdot \|$, which is a measure of similarity in (3.6) compares members of the data to each other indirectly via distance between them and the cluster centers.

Optimal fuzzy c-partitions of X are defined as part of pair $(\hat{U}, \hat{\underline{V}})$ which solves the problem.

Minimize:

$$J_m(U, \underline{V}) \text{ over } M_{fc} \otimes R^{cs} \qquad (3.7)$$

which is also called the generalised minimum variance partitioning problem associated with X. It can be seen that for $U \in M_c$, $J_m = J_1 \; \forall \; 1 \leq m < \infty$, and J_m reduces to the classical WGSS criterion function. Detailed discussions on the merits and drawbacks of J_1 and its extension J_m can be found in Wishart [111], Ling [112], Dunn [42, 45], and Bezdek [21,51]. Partitions obtained as part of solutions for (3.7) have been shown to be related to a well defined type of hard, compact, well separated (CWS) clusters for X in [42]. The necessary conditions for solutions of (3.7) are derived in [44] for the class of functionals in (3.6) whose norms are differentiable (e.g., inner product induced norms). It has been shown that for $m > 1$ and $\underline{x}_i \neq \underline{v}_k \; ; \; \forall$ i, k , the necessary conditions in order for $(\hat{U}, \hat{V})$ to be a local solution for (3.7) are:

$$\hat{u}_{ki} = \left[\frac{1}{\sum_{j=1}^{c} \left(\frac{\|\underline{x}_i - \underline{v}_k\|}{\|\underline{x}_i - \underline{v}_j\|} \right)^{\frac{2}{m-1}}} \right] ;$$

$$\forall \; 1 \leq k \leq c \; ; \; 1 \leq i \leq n \qquad (3.8a)$$

$$\hat{\underline{v}}_k = \left[\frac{\sum_{i=1}^{n} [u_{ki}]^m \, \underline{x}_i}{\sum_{i=1}^{n} [u_{ki}]} \right] ; \forall \; 1 \leq k \leq c \qquad (3.8b)$$

$$\hat{u}_{ki} \in (0, 1) \quad ; \forall\, 1 \leq k \leq c \; ; \quad 1 \leq i \leq n \qquad (3.8c)$$

The complete details for m = 1 and the singular case $\underline{x}_i = \hat{\underline{v}}_k$ for some 'i' and 'k' may be found in [42,44]. For m = 1, the requirement (3.8a) is replaced by a nearest neighbourhood assignment rule and $U \in M_c$ is necessarily hard, the cluster centers in (3.8b) are merely the centroids of the hard subsets in U, and the resultant algorithm is essentially the 'hard ISODATA' process of Ball and Hall [43]. For m > 1, equations (3.8) define the conditions for the 'fuzzy ISODATA' algorithm which has been used in this work. The major drawback with hard ISODATA process is that it may show the presence of clusters even though there may not be any natural groups present in the data set. The fuzzy ISODATA clustering process largely overcomes this difficulty as reported by Dunn [42, 44].

After this review of fuzzy sets and fuzzy clustering, we now proceed to describe how the medicine inventory classification problem can be solved by abstraction of the medicines into feature vectors representing their

importance and how this can be used to effect selective inventory management for improved economic control of inventory.

3.5.2 Importance through Feature Vectors

In order to reveal the natural classes present in the medicine inventory by Fuzzy ISODATA process, the relative importance of an item has to be expressed in a vector form. The first step in this direction will be to objectively spell out which features or characteristics of an item contribute to its importance and how can they be measured. A suggested list of features for the vectorial representation of importance of medicines is given below. As stated earlier, we shall use the notation x_{ij} to represent the j-th characteristic of the i-th item. For ease in comparison and clustering, the features have been defined in such a way that all values lie between 0 and 1.

(1) Annual Rupee Usage Component

The first important characteristic is the annual rupee usage of an item. The annual rupee usage of an item is obtained by multiplying the annual consumption in units by the unit cost of the item. An item with higher annual rupee usage is more important as it entails higher investment. Let,

C_i = cost per unit of item 'i',

A_i = average yearly consumption of item 'i' in units,

n = total number of types of items in inventory,

x_{i1} = annual rupee usage component of item 'i'.

Then,

$$x_{i1} = \frac{C_i A_i}{\text{Max}\left\{C_i A_i \; ; \forall\, i = 1 \text{ to } n\right\}} \; ; \forall\, i = 1(1)\, n$$

(2) Expiry Time Component

Very often problems arise when due to lack of attention not enough time is left before the medicines can be utilised in order to prevent the loss due to expiry. An item with shorter expiry time deserves more attention than an item with relatively larger expiry time.

Let,

E_i = expiry time for item 'i' in months,

x_{i2} = expiry time component of feature vector of item 'i',

$E_{max} = \text{Max}\left\{E_i \; ; \forall\, i = 1(1)\, n\right\}$.

Then,

$$x_{i2} = \frac{(E_{max} - E_i)}{E_{max}} \; ; \forall\; i = 1(1)\; n$$

(3) Life-Saving Importance Component

This characteristic may be compared with 'shortage cost' in industrial and commercial establishments. Some medicines are such that if they are not available in time, the consequences can be serious to the extent of causing permanent impairment or even death of the patient. On the other hand, there are several other medicines like tonics, vitamin tablets and the like, which do not matter much if there is a stock-out as their nonavailability will not cause any serious damage. It is difficult to measure this characteristic and therefore the values have to be assigned to this component only intuitively.

Let this component be denoted by x_{i3} and we assign values:

x_{i3} = 0 ; if the item is unimportant,

x_{i3} = 0.5 ; if some damage may possibly be caused due to its nonavailability,

x_{i3} = 1.0 ; if the item is very important.

This subjective assignment of values is justifiable on the ground that the fuzzy ISODATA algorithm used for clustering groups similar items together and if two items are of equal importance they will be put in the same group and will be controlled by the same procedure.

(4) Substitutability Component

Most medicines can be substituted by others in several situations. If substitutes are always readily available we may not like to give as much attention to an item as we will give to an item which has no substitute. This characteristic is also difficult to measure and the values have to be intuitively assigned.

Let X_{i4} represent the substitutability feature of item 'i'. We assign the following values:

$x_{i4} = 0$; if the item can always be substituted,

$x_{i4} = 0.5$; if the item can be sometimes substituted,

$x_{i4} = 1.0$; if the item can never be substituted.

A similar situation is present in departmental stores where for several items such as soaps, tooth-pastes, shaving creams etc, one brand may be acceptable to the customer for another.

(5) Market Availability Component

There may be some items which are difficult to procure from the open market when needed such as baby foods and some type of injections. This is particularly true for grocery stores where several items are always in short supply. This component can be measured using the availability ratio relationship given as,

$$x_{i5} = \frac{\text{Number of times item not available from supplier}}{\text{Total number of times purchase orders made}} ;$$

$$\forall\ i = 1(1)\ n$$

where,

x_{i5} = market availability component of item 'i'.

If past records are difficult to search, a simple measure based on subjective judgement may be used. We assign values to x_{i5} as:

x_{i5} = 0 ; if the item is always available,

x_{i5} = 0.5 ; if the item is only sometimes not available,

x_{i5} = 1.0 ; if the item is difficult to procure.

(6) Stock-Out History Component

It is desirable to include an indicator of the efficiency of past and existing control procedures for deciding the importance of a medicine. If the earlier procedures were incompatible and have resulted in frequent stock-outs, the item needs to be promoted to the class of more important items because it requires better control. The stock-out history component, x_{i6}, for the i-th item is expressed as,

$$x_{i6} = \frac{\text{Number of times the item found out of stock}}{\text{Total number of checks made}} ;$$

$$\forall\ i = 1(1)\ n .$$

As an alternative the values 0, 0.5 and 1.0 may be assigned to this component for items which were practically never out of stock, sometimes out of stock and frequently out of stock in that order.

(7) Storing Method component

Some of the medicines, particularly injections, require special storing methods like low temperature, protection from sun light and humidity, etc. The stocks of these medicines should not be allowed to pile up to the extent that it becomes difficult to accommodate them.

Let this component be denoted by 'x_{i7}'. We assign the values:

x_{i7} = 0 ; if no special care is needed in storing,

x_{i7} = 0.5; if some special care has to be exercised,

x_{i7} = 1 ; if special care is required.

(8) Storage Space Component

Sometimes when there is a limitation on the total space available for storing, particular care should be taken in maintaining stocks of items occupying too much space in order to accommodate other items. Though this characteristic may not be of much consequence in many practical situations, yet it needs to be incorporated for the sake of generality.

Let us denote this component by 'x_{i8}'. Then,

$$x_{i8} = \frac{\text{Space occupied by item 'i' on the average}}{\text{Maximum of the spaces occupied by all items}} ;$$

$$\forall \; i = 1(1) \; n.$$

One can determine x_{i8} by determining the space requirements of the various items. An alternative proposition is to group the items into three categories and assign values of 0, 0.5 and 1.0 depending upon whether the storage space requirements are low, medium or high for the item.

(9) Lead Time Component

The procurement lead time adds significantly to our feeling of importance for an item. Needless to say, items with higher lead time need greater attention. Let,

x_{i9} = lead time component of item 'i',

T_i = lead time of item 'i',

$T_{max} = \text{Max} \{ T_i \; ; \forall \; i = 1(1) \; n \}$.

Then,

$$x_{i9} = \frac{T_i}{T_{max}} ; \quad \forall \; i = 1(1) \; n \quad .$$

(10) Lead Time Variability Component

If the procurement lead time for an item is highly varying it deserves more attention and larger safety stocks. Let,

$x_{i,10}$ = lead time variability component of item 'i',

$\sigma^2_{T_i}$ = variance of lead time for item 'i',

Then,

$$x_{i,10} = \frac{\sigma^2_{T_i}/T_i}{\text{Max}\left\{(\sigma^2_{T_i}/T_i);\ \forall i = 1(1)n\right\}};\ \forall i = 1(1)\ n.$$

If it is difficult to calculate T_i and $\sigma^2_{T_i}$, the values 0, 0.5 and 1.0 may be assigned as an alternative to items with low, medium or highly varying lead times.

(11) Demand Variability Component

If the demand for an item is highly fluctuating, it needs a better forecasting method to account for all the factors due to which the variations occur and to track the changes. One of the measures may be the ratio of variance with respect to mean demand. Let,

$x_{i,11}$ = demand variability component,

$\bar{D}_i$ = mean demand of item 'i' per period,

$\sigma^2_{D_i}$ = variance in demand per period.

Then,

$$x_{i,11} = \frac{(\sigma^2_{D_i} / \bar{D}_i)}{\text{Max}\left\{\sigma^2_{D_i}/\bar{D}_i \; ; \forall\, i = 1(1) \;\; n\right\}} \; ; \forall\, i = 1(1)\; n.$$

(12) Quantity Discount Component

The bulk purchase quantity discount may vary from item to item and manufacturer to manufacturer depending upon their marketing policies. Items for which higher discounts are available deserve more attention and care should be taken to use these discounts to advantage. Let,

$x_{i,12}$ = quantity discount component for item 'i',

P_i = percent quantity discount available for item 'i',

C_i = cost per unit of item 'i',

A_i = annual consumption of item 'i'.

Then,

$$x_{i,12} = \frac{P_i C_i A_i}{\text{Max}\left\{P_i C_i A_i \; ; \forall\, i = 1(1)\; n\right\}} \; ; \forall\, i = 1(1) \;\; n \; .$$

The above list of features may not be exhaustive but it gives an idea regarding how to go about defining and expressing the feeling of importance for a medicine through feature vector .

(x) Total space occupied by 'DRISTAN' tablets is 'too much'.

(xi) Procurement lead time is 40 days with a variance of 10 days and maximum lead time for any item is 100 days.

(xii) Maximum value of variance to lead time ratio for any item is 0.5.

(xiii) Ratio of variance to mean demand = 0.4, and the maximum value of this ratio for any item is 1.0.

(xiv) Maximum quantity discount available for 'DRISTAN' is 20 percent, and the maximum value of the factor $P_i C_i A_i$ for any item is Rs. 3000/-.

From the above data and the definition of various feature vector components, the feature vector of 'DRISTAN' is:

$$\underset{\sim}{x} = \{0.25,\ 0.75,\ 0.5,\ 0,\ 0.5, 0, 0.5, 1.0, 0.4, 0.5, 0.4, 0.5\}$$

After conversion of the information available about the medicines in stock into feature vectors, the next step is to classify the items into various importance groups.

3.5.3 Classification by Fuzzy ISODATA Process

As we have already stated, conditions (3.8) together with convergence criterion and tie breaking rules form the basis of a process of simple iterative optimisation of J_m, defined in equation (3.6), called fuzzy ISODATA which has been used here to form clusters from the input of feature vector data.

The algorithm starts with a set of 'c' initial centroids. (The next section deals with the selection of these centroids). In every iteration the membership grades of each item with respect to every cluster are determined and the centroids are recalculated. The iterations are stopped when two successive sets of centroids are very close to each other. The steps in the algorithm are as follows:

(1) Start with 'c' initial centroids $\underset{\sim}{v}_1, \underset{\sim}{v}_2, \ldots, \underset{\sim}{v}_c$.

(2) Set i = 1.

(3) Consider the feature vector $\underset{\sim}{x}_i$ of item 'i'.

(4) If $\underset{\sim}{x}_i$ is equal to any of the centroids, say $\underset{\sim}{v}_j$, compute the membership grades of $\underset{\sim}{x}_i$ with respect to the clusters by:

$$u_j(\underset{\sim}{x}_i) = 1 \text{ ; for } i = j \text{ , and}$$

$$u_j(\underset{\sim}{x}_i) = 0 \text{ ; for } \forall\, j \neq i.$$

If $\underset{\sim}{x}_i$ is not equal to any of the centroids, then:

$$u_k(\underset{\sim}{x}_i) = \left[\frac{1}{\sum_{l=1}^{c} \left(\frac{\| \underset{\sim}{x}_i - \underset{\sim}{v}_k \|}{\| \underset{\sim}{x}_i - \underset{\sim}{v}_l \|} \right)^{\frac{2}{m-1}}} \right] \text{ ; for } \forall\, k = 1(1)\, c. \qquad (3.9)$$

where $\|\cdot\|$ is a norm induced by the weighted inner product

$$\left\langle \underset{\sim}{x}, \underset{\sim}{x} \right\rangle = \underset{\sim}{x}^t A \underset{\sim}{x} \quad \text{on } R^s, \text{ and}$$

$$A = [\, \text{diag} \left(\frac{\sigma_1^2}{w_1}, \ldots, \frac{\sigma_s^2}{w_s} \right)]^{-1}$$

$$= \text{diag} \left(\frac{w_1}{\sigma_1^2}, \ldots, \frac{w_s}{\sigma_s^2} \right),$$

$\sigma_1^2, \ldots, \sigma_s^2 \Rightarrow$ variances of the feature vector components,

$w_1, \ldots, w_s \Rightarrow$ normalised weightages given to various feature vector components,

s = number of feature vector components.

Upon substitution of the norm in (3.9), we get,

$$u_k(\underset{\sim}{x}_i) = \frac{\left[\dfrac{1}{\sum_{j=1}^{s} w_j \left(\dfrac{x_{ij} - v_{kj}}{\sigma_j} \right)^2} \right]^{(1/(m-1))}}{\sum_{l=1}^{c} \left[1 \Big/ \sum_{j=1}^{s} w_j \left(\dfrac{x_{ij} - v_{lj}}{\sigma_j} \right)^2 \right]^{\left(\frac{1}{m-1}\right)}} ;$$

$$\forall\, k = 1(1)\, c \qquad (3.10)$$

(5) Set $i = i + 1$

(6) If $(i > n)$ go to next step, otherwise go to step (3), where 'n' is the number of feature vectors.

(7) Calculate new centroids by:

$$\hat{\underset{\sim}{v}}_k = \frac{\sum_{i=1}^{n} [u_k(\underset{\sim}{x}_i)]^m \, \underset{\sim}{x}_i}{\sum_{i=1}^{n} [u_k(\underset{\sim}{x}_i)]^m} \qquad (3.11)$$

(8) If,

$$\left\{ \sum_{j=1}^{s} (\hat{v}_{kj} - v_{kj})^2 \right\}^{1/2} < \varepsilon \quad ; \quad \text{for } \forall\, k = 1(1)\, c, \text{ stop ;}$$

Otherwise put $\underset{\sim}{v}_k = \hat{\underset{\sim}{v}}_k$ and go to step (2) where 'ε' is a small quantity whose value depends upon the accuracy desired.

When the process is complete, final centroids together with the membership grades of items in each cluster are printed out. As the algorithm treats all the elements in the feature vector as continuous variables, the centroids do not represent any real item. For a given set of weights, the fuzzy clustering process is repeated for several values of 'c' and 'm'. It should be noted that for any item the membership grades sum to unity over all the clusters. Figure (1) depicts the logic of Fuzzy ISODATA process. At this moment two questions remain unanswered. Firstly how should the initial centroids be chosen and second what would be the best choice for 'c' and 'm'. In the next section we will try to answer these questions.

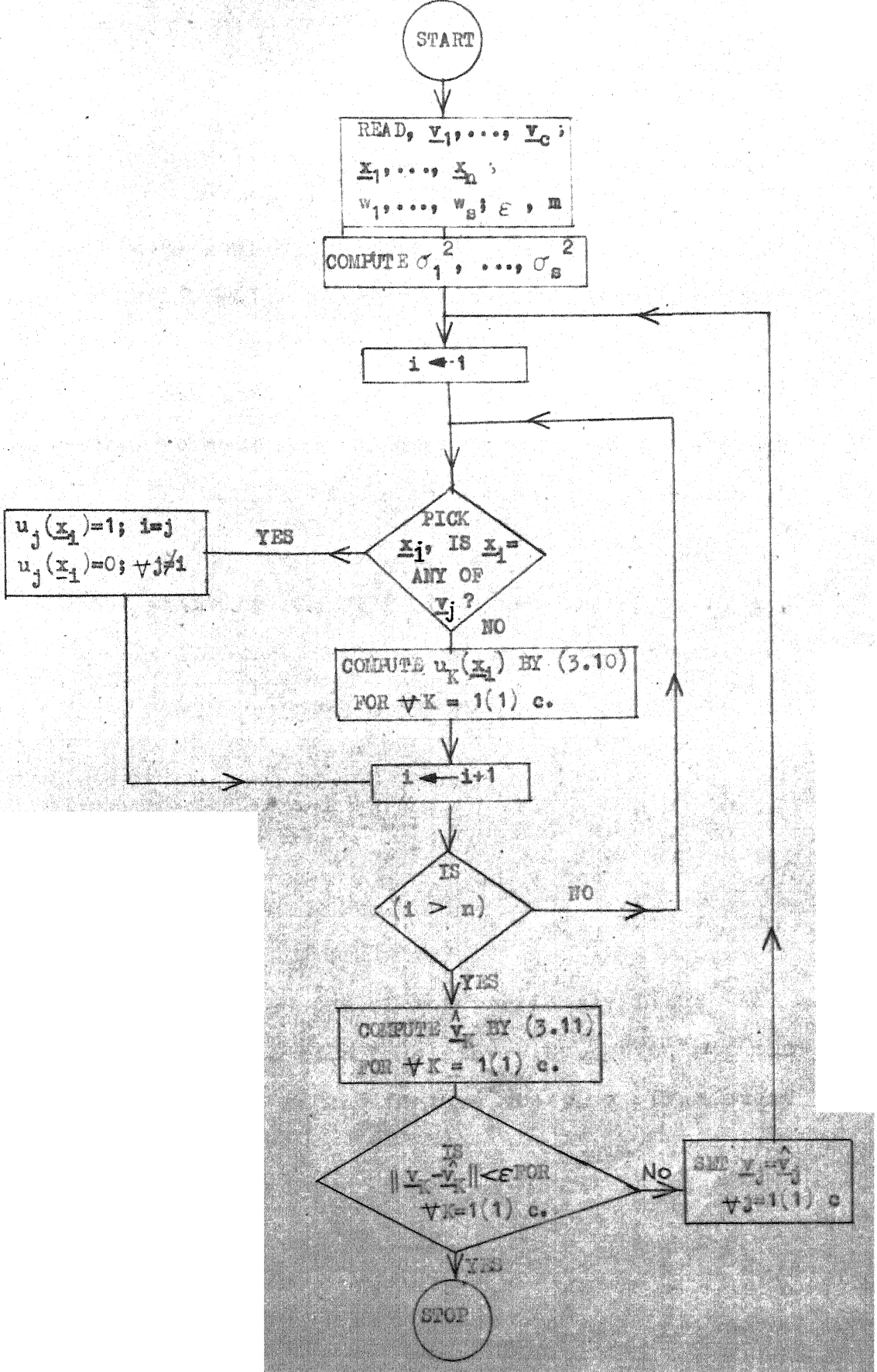


FIG. 1

LOGIC OF THE FUZZY ISODATA PROCESS.

3.5.4 Choosing the Initial Centroids

The fuzzy ISODATA is sensitive to the choice of initial centroids [53]. So far no absolutely satisfactory method has been reported for this purpose. Anderberg [30] has given several methods for choosing the initial centroids. In this work, we have used a method suggested by Ball and Hall [113]. The method can be summarised in the following steps:

(1) Start with the set of feature vectors $\underset{\sim}{x}_1, \ldots, \underset{\sim}{x}_n$.

(2) Fix the first centroid $\underset{\sim}{v}_1$ as the mean feature vector whose components are given by:

$$v_{ij} = \frac{\sum_{i=1}^{n} x_{ij}}{n} \quad ; \forall\, j = 1(1)\, s$$

and set K = 1.

(3) Set i = 1.

(4) If $\underset{\sim}{x}_i$ is same as any of the previously fixed 'K' centroids go to step (6); otherwise, calculate the distance 'd_{ip}' of the feature vector $\underset{\sim}{x}_i$ from every centroid by:

$$d_{ip} = \left\{ \sum_{j=1}^{s} w_j \left(\frac{x_{ij} - v_{pj}}{\sigma_j} \right)^2 \right\}^{\left(\frac{1}{m-1}\right)} \quad ; \forall\, p = 1(1)\, K.$$

(5) If $[d_{ip} \geq \theta \; ; \forall \; p = 1(1) \; K]$, set $\underset{\sim}{y}_{K+1} = \underset{\sim}{x}_i$, $K = K+1$, and go to step (3). where θ is a conveniently set quantity denoting that all initial centroids should be separated from each other atleast by this distance.

(6) If $(i = n)$ and no new centroid could be found go to next step, otherwise set $i = i+1$ and go to Step (4).

(7) If $(K = c)$ stop. If $(K < c)$ change θ to $(\theta - \lambda)$; If $(K > c)$ change θ to $(\theta + \lambda)$ and go to Step (2), where λ is a small quantity.

The centroids thus generated for various values of 'c' are taken as input to the fuzzy ISODATA. The next section deals with finding the optimal number of clusters (c) and the effect of exponent (m) on clustering.

A logic flow diagram for the selection of initial centroids is given in Fig. (2).

3.5.5 Measures of Partition Quality

For the first time Bezdek [22] has suggested a method for systematically finding the number of clusters in X. Though so far only a limited number of experiments have been conducted with this method, we have found it quite useful during the course of our work. The method is based on the concept of 'quality' associated with fuzzy clusters.

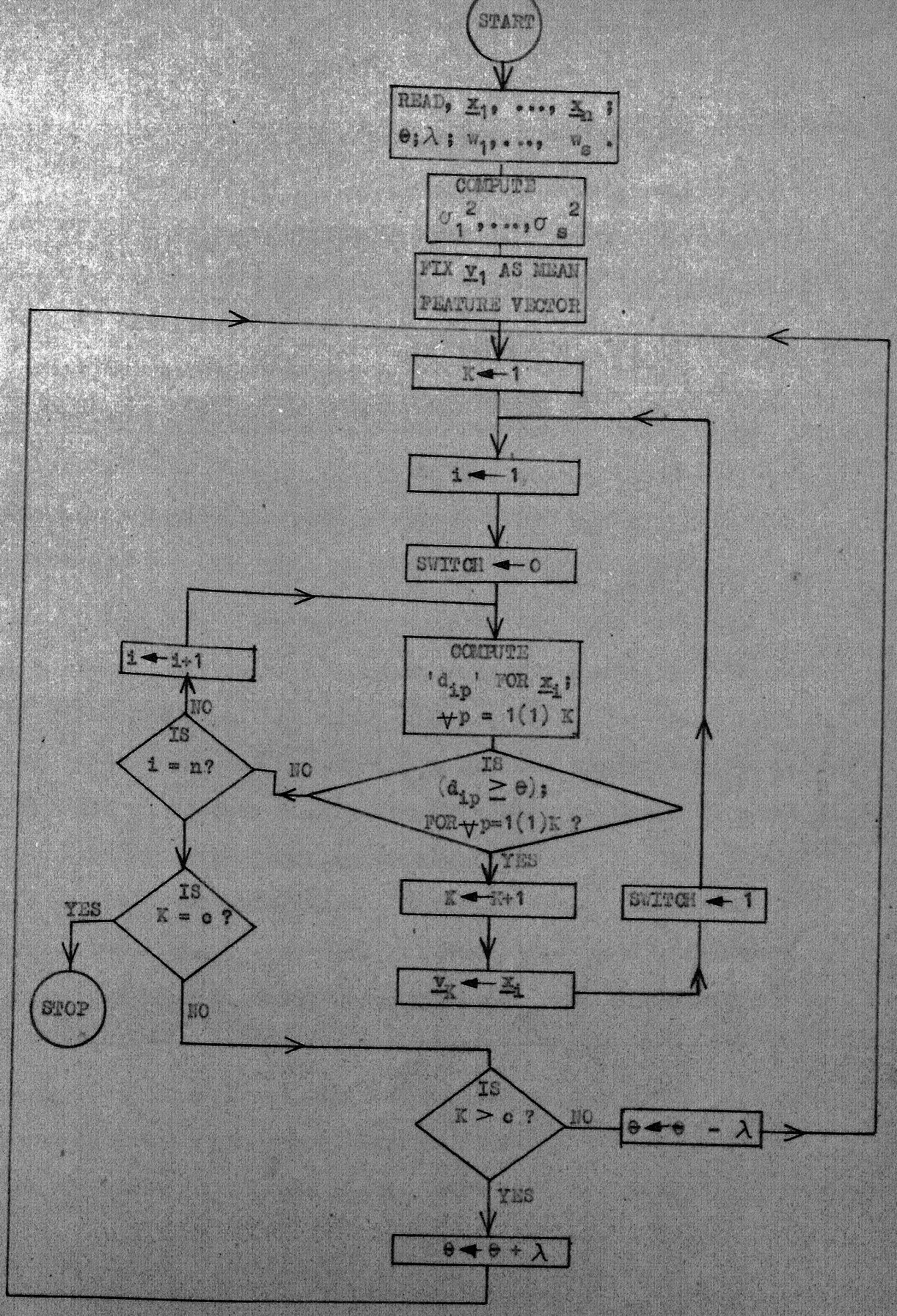


FIG. 2

SELECTION OF INITIAL CENTROIDS.

Irrespective of the procedure used for clustering we may have to answer questions like: does the algorithmic output really correspond to structure in X? More generally, does it correlate with natural clusters in the real model that X represents? In all but trivial instances [22], the answer to these questions is easy, we simply do not know. The only criterion capable of evaluating algorithmic clusters is undisputable a priori labelling (i.e. pre-partitioned samples), in which case there is no problem to solve.

The difficulty arises because 'cluster' becomes a meaningful term only in a well defined sense usually deriving from the nature of the data, and of course the model we choose to represent it with. It appears that the heuristic approach of finding an algorithm capable of reliable decomposition of large training samples from the process under study is the only feasible alternative when dealing with hard clustering techniques. On the other hand, the fuzzy model provides some added information about structure in X which seems useful in evaluating the relative merits of competing classifications independent of the clustering procedure used to obtain them. Bezdek [22] has suggested two measures of evaluating the partition quality.

The first measure is via the 'partition coefficient' $F_c(U)$ which can be defined as follows:

$$F_c(U) = \text{trace } (UU^t)/n = \frac{\|U^2\|}{n} \; ; \forall U \in M_{fc} \quad (3.12)$$

The values of $F_c(U)$ are inversely proportional to the average content of fuzzy intersections in U and it intuitively provides a measure of the uncertainty an algorithm experiences when assigning membership values to individuals in X. It seems plausible that an algorithm yielding fuzzy c-partitions with higher value of F_c is more successfully decomposing X than one whose partitions are inherently 'fuzzier'.

A second approach to this question arises from similarities to the notion of uncertainty usually used in information theory. Probabilistic measures of uncertainty in connection with classification models have been investigated by Estabrook [116], Orloci [117], and Wallace and Boulton [118]. Ruspini [38] and Deluca and Termini [13] have discussed the notion of entropy in the context of fuzzy sets. Though there are philosophical distinctions between probabilistic and fuzzy uncertainties , the formal similarity between them suggests exploiting some mathematical properties known in information theory to describe statistical uncertainty. The 'normalised classification entropy' in U, denoted by $H_c(U)$, based on the Shannon's entropy

function, suggests using a function:

$$H_c(U) = \frac{\left\{ - \sum_{i=1}^{n} \sum_{l=1}^{c} u_{il} \log_a (u_{il}) \right\}}{n} \qquad (3.13)$$

with $u_{il} \log (u_{il}) =, 0$, if $u_{il} = 0$.

Although the functional forms of F_c and H_c are quite different, the relation between them can be shown to be [22] as follows.

$$F_c (U) = 1 \Leftrightarrow H_c(U) = 0 \Leftrightarrow U \varepsilon M_c \text{ is hard} \qquad (3.14)$$

$$F_c (U) = \frac{1}{c} \Leftrightarrow H_c(U) = \log_a c \Leftrightarrow U = [1/c] \qquad (3.15)$$

Further, Bezdek [22] has proved that the upper and lower bounds on $H_c(U)$ are:

$$1 - F_c(U) < \left(\frac{H_c(U)}{\log_a e} \right) < \frac{1}{2} (c - F_c (U)) \qquad (3.16)$$

For a = e this reduces to:

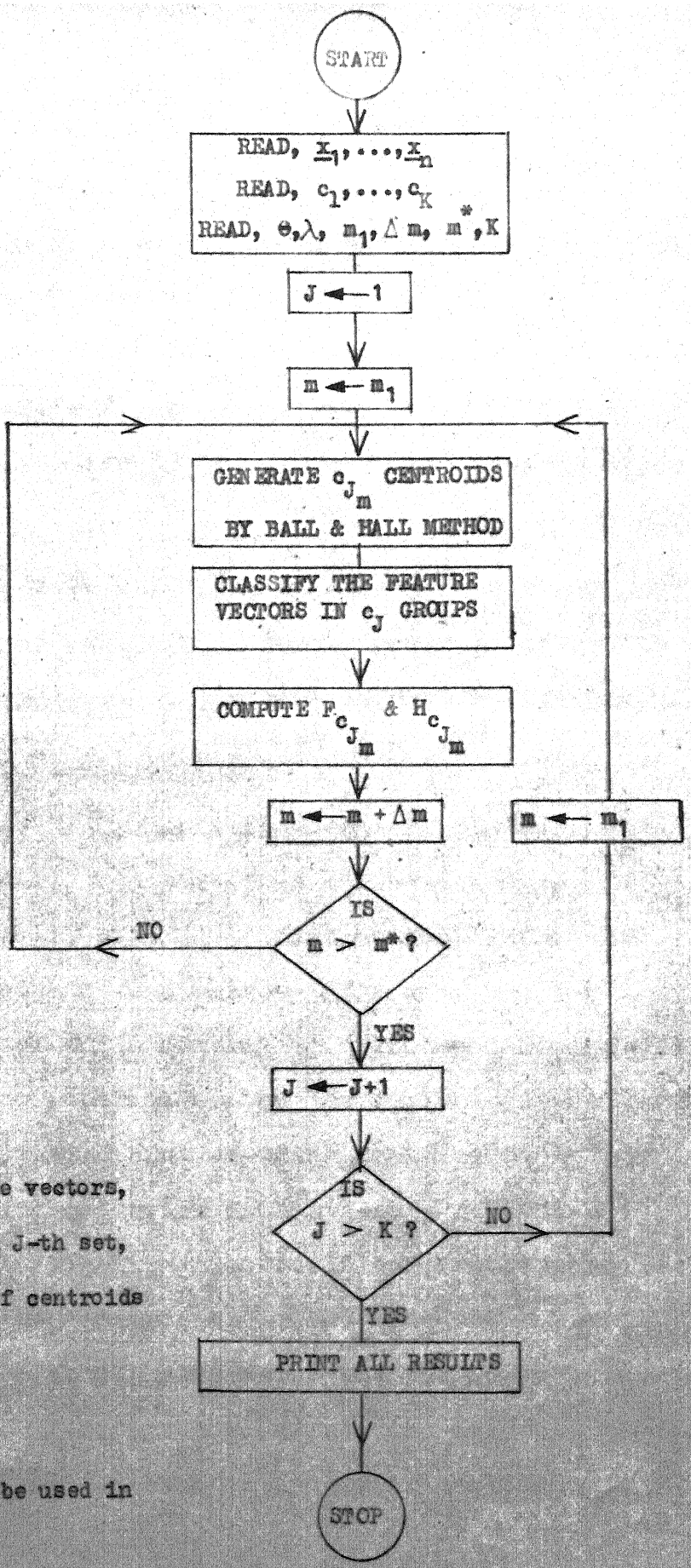
$$1 - F_c (U) < H_c(U) < \frac{1}{2} (c - F_c(U)) \qquad (3.17)$$

Equations (3.14,15) suggest that the equi-membership partition $\tilde{U} = [\frac{1}{c}]$, i.e. $\tilde{u}_{il} = \frac{1}{c}$;$\forall$ i, l, is the fuzziest or worst one can do; on the other hand, the ideal situation occurs when the structure in X is so distinct that a fuzzy algorithm recommends a hard c-partitioning of X. Maximising

'F_c' over different fixed points of fuzzy algorithms minimises the total content or overlap in pairwise fuzzy intersections; equivalently, minimising 'H_c' over the same choices maximises the information extracted from U. These measures serve as a relative indication of the uncertainty an algorithm experiences in trying to assign memberships to the vectors in X.

Numerical evidence [22] indicates that 'H_c' is more sensitive than 'F_c' in ranking U's; this is due to the fact that the slope of the logarithmic curve on most of (0, 1) is much steeper than that of a parabola. In general the clustering strategy has been to try fuzzy ISODATA for clustering for a given set of weights, over several values of 'c' and 'm' and regard that c-partitioning of X as optimal which yields minimum value of 'H_c'. The scheme has been shown in Fig. (3).

In the present investigation of medicine inventory both the measures of partition quality were tried. It was found that both of them suggest the presence of three partitions. The detailed results regarding these are presented in Chapter IV. The first group consists of medicines which are important from all considerations, second group has medicines which have low annual rupee usage and high expiry time but are otherwise important and the third group comprises of unimportant medicines. For assigning the medicines to various



Legend:

n = total number of feature vectors,

c_J = number of centroids in J-th set,

K = total number of sets of centroids to be tried,

m = weighing exponent,

Δm = small increment in m,

m^* = maximum value of m to be used in analysis.

FIG. 3

GENRAL PROCEDURE FOR ITEM CLASSIFICATION.

clusters, the 'maximum membership grade rule' has been followed. Some other rules which use a higher cut-off value are also available. However, to avoid misclassifications it is better to present the membership grade distribution information as it is and then make suitable transfers within various classes if necessary.

The next task before us is to select or devise suitable ordering schemes for various classes of medicines and the remaining part of the chapter is devoted to this.

3.6 Selecting the Ordering System

In inventory models we attempt to isolate variables and parameters and try to understand the behaviour of inventories. Further, we seek to find optimal values of these variables under a wide variety of conditions and different degrees of model complexity. Then the management selects an inventory control system for implementation based on a number of criteria, many of which conflict with each other. An important criterion in a system's selection is its adaptability to existing procedures and regulations. This is necessary to minimise the resistance encountered while changing over to the new system and the implementational costs and delays. Therefore a look into the existing procedures for controlling medicine inventory in hospitals is desirable for evolving the new system.

The present regulations in most hospitals ask for maintaining perpetual stock records for each item. Medicines are issued to compounders from stores, as and when needed, and the transactions are entered in a ledger. When the stock of an item has depleted to some pre-set reorder point, a replenishment order for a suitable quantity is placed. At present, in most of the hospitals, no scientific method is used to calculate the reorder level and reorder quantity and the decisions regarding these are taken mostly on the basis of intuition.

From the above, it is clear that a continuous review reorder point system will be readily adaptable to present regulations. Here we have two alternatives. We can either opt for a conventional fixed limit control system or choose an adaptive control limit system. This decision is not a difficult one to make as the superiority of adaptive limits for inventory control is now more than established. Eilon and Elmaleh [114] on having simulated about 200 cases over 500 periods, reported 30 percent reduction in inventory costs over fixed limit controls by using adaptive control procedures. Trux [55] has also stated that 30-40 percent savings occur by using adaptive limits.

Though we have seen that the 'once and for all' determination of control parameters based on historical data is clearly unsatisfactory, the analytical treatment of mathematical models incorporating the real life features of stochastic demand and lead time is too complicated to be useful for practical applications. Wagner et al. [115] have demonstrated the computational difficulties involved in determining control limits in such cases.

In view of the above, we have suggested a method here which is based on the works of Buchan and Koeningsberg [2] and DeMattis [3]. This method for computing adaptive control limits is simple and straight forward, and it takes into account seasonal fluctuations and trends in demand. It, however, does not seem to be economical for the class of 'unimportant' items and therefore fixed limit controls have been used for these items. Before the details of the procedures are described, it is desirable to have an overview of the system. The next section deals with this.

3.7 An Overview of the System

The general procedure will be to first locate 'c' initial centriods by Ball and Hall's method. Starting with these initial centroids, we classify the items for several values of 'c' and 'm' and accept that classification which yields best values of the measures of quality, 'H_c' and 'F_c'.

This classification needs to be done only once in a year or when there are a large number of additions or deletions in the list of the items. This information together with the demand, cost and lead time data is taken as input to the program for computing lot sizes and reorder points for specified service levels. The program uses different algorithms for computing these depending upon the class to which the item belongs to and gives a detailed report for the purpose of control. The procedure has been shown in Fig.[4].

The reorder level should be noted on the same page of the ledger which is being used for keeping the stock record for the item. When the stock falls below the reorder level, then the program which works on independent ordering policies may be run for only those items which need to be reordered.

Though in its present form the system is only semi-computerised, it can be extended for automatic data processing. The present card files can be replaced by storage devices on a computer and the transactions may be directly entered. The reorder level information can be stored side by side and whenever a transaction occurs, a computer check can be made to find whether an order needs to be placed.

Having described the general logic of the system, we shall now elaborate different procedures used for

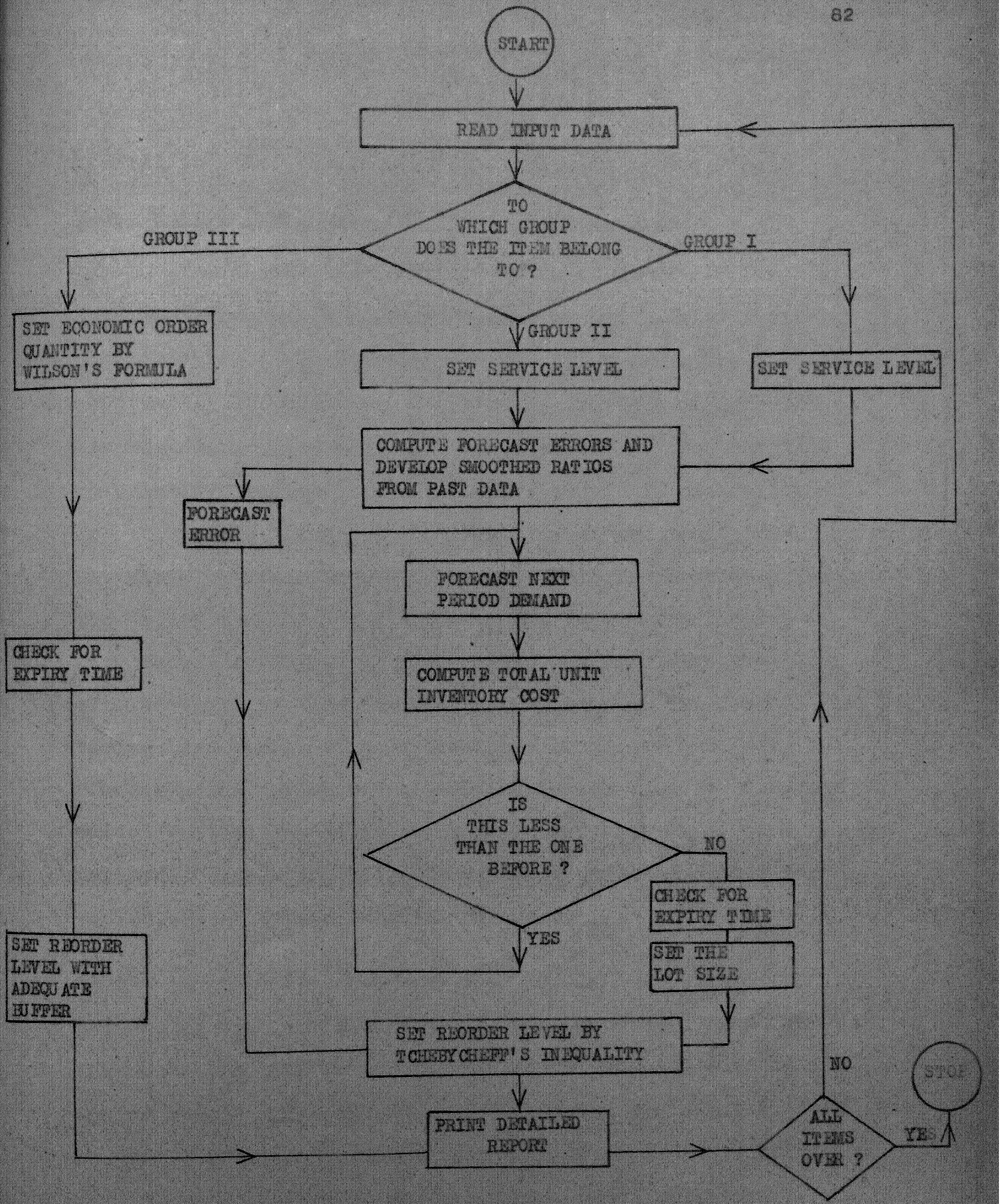


FIG. 4

GENERAL LOGIC FLOW DIAGRAM FOR COMPUTING LOT SIZES AND REORDER LEVELS.

controlling the inventory of three classes of medicines.

3.8 Control Procedure for 'Class I' Medicines

The first class consists of those medicines which are important from all considerations. Keeping in view their high importance, an adaptive control limit procedure has been used. The procedure essentially consists of forecasting the monthly requirements for each item and at the same time calculating the total unit inventory cost. The calculations are terminated when the lot size for minimum unit total inventory cost has been determined. This is checked against the expiry time of the medicine. The forecast error is also accumulated side by side. This together with the lead time variance is used to find the variance of the demand during lead time. This information is then used to set the reorder point for specified service levels by using Camp and Meidel's extension of Tchebycheff's inequality. The control parameters are recalculated when a new order is placed for replenishment by updating the forecast.

3.8.1 The Forecasting Scheme

The ordering decisions are based on the demand forecast for each item. The demand for medicines exhibits seasonality and an upward trend in general. The trend can be attributed to the population growth and increasing medical facilities.

Seasonality is due to the climatic changes. A forecasting method suggested by Buchan and Koenigsberg [2] matches with this situation and is economical to use for a large number of types of medicines. The following notation is used for describing the forecasting procedure. For an item 'i', let,

D_{ijl} = demand in period 'j' of year 'l',

K_i = number of years for which past demand data is available,

N = number of periods in a year ($N = 12$ in this case),

$\bar{F}_{it}$ = demand forecast for period 't',

α = smoothing constant,

τ_i = trend per period in demand,

R_{ij} = base series ratio for period 'j',

$\bar{R}_{ij}$ = smoothed ratio upto period 'j',

B_{ij} = base series value for period 'j',

N_{ic} = number of periods of current year for which demand data is available.

The steps are:

[1] Compute the trend per period in the demand for the medicine by:

$$\tau_i = \frac{\sum_{i=1}^{N} D_{ijK_i} - \sum_{i=1}^{N} D_{ijl}}{N\{(K_i - 1)\, N\}} \quad .$$

[2] Develop the seasonal base by eliminating the overall monthly trend 'τ_i' from the average monthly demand series as follows:

$$B_{ij} = \frac{\sum_{l=1}^{K_i} D_{ijl}}{K_i} + \left\{\left(\frac{N}{2} - 0.5\right) - j\right\} \tau_i \; ;$$

$$\forall\, j = 1(1)\, N \; .$$

[3] Compute the smoothed ratios by:

$$R_{ij} = \frac{D_{ijK_i}}{B_{ij}} \; ; \quad \forall\, j = 1(1)\, N,$$

$$\bar{R}_{il} = R_{il} \; ,$$

$$\bar{R}_{ij} = \alpha\, R_{ij} + (1 - \alpha)\, \bar{R}_{i,j-1} \; ; \quad \forall\, j = 2(1)\, N.$$

If $N_{ic} \neq 0$, the smoothed ratio series is updated upto the N_{ic}-th period of the current year.

[4] The demand forecast of the L_i-th period ahead is given by,

$$\bar{F}_{i,(t+L_i)} = \bar{R}_{i,t-1}\, B_{(t+L_i-1)} + L_i\, \tau_i$$

The demand forecast is taken as input to the lot sizing model which is described next.

3.8.2 Lot-Sizing Procedure

A dynamic lot-sizing procedure has been used for the 'Class-I' medicines to determine the order quantities. It is a modified version of the Part Period Algorithm [3]. The modification is made in calculating the cost which is calculated here on the basis of average inventory since no depletion is assumed immediately upon arrival of the medicine in stock. Further, in case of medicines the prices and carrying costs can be assumed constant over a considerable period of time and, therefore, the dynamic lot-sizing model used here assumes only the demand as variable. The following notation will be used in describing the model:

For any medicine 'i', let,

$C_{im'}$ = total cost per unit if the lot covers requirements upto m'-th period,

h_{is} = stock-holding cost per unit per period in Rs.,

P_i = price per unit in Rs.,

p_{ic} = carrying cost in percent,

C_{is} = set-up cost per order in Rs.,

N = number of periods per year,

$\bar{F}_{ij}$ = demand forecast for period 'j' in units,

E_i = expiry time for themedicine in periods,

A_i = annual demand of the medicine in units,

Q_i^* = optimal lot-size in units.

It can be noticed here that the stock-holding cost per unit per period is given by:

$$h_{is} = \frac{P_i \, p_{ic}}{100(\bar{N})} \text{ Rs./period/unit.}$$

The lot-sizing procedure can be summarised in the following steps:

(1) Compute 'h_{is}', set m' = 1, forecast $\bar{F}_{il}$ and find

$$C_{il} = \frac{C_{is} + (h_{is} \, \bar{F}_{il}) \, (0.5)}{\bar{F}_{il}}$$

(2) Set, m' = m' + 1

(3) Forecast the demand $\bar{F}_{im'}$ for period 'm".

(4) Compute the total cost per unit by:

$$C_{im'} = \frac{C_{is} + h_{is} \sum_{j=1}^{m'} \bar{F}_{ij} \, (j - 0.5)}{\sum_{j=1}^{m'} \bar{F}_{ij}}$$

(5) If, $C_{im'} > C_{i,m'-1}$, take $C_i^* = C_{i,m'-1}$,

$Q_i^* = \sum_{j=1}^{m'-1} \bar{F}_{ij}$ and stop. If not so go to Step (6).

(6) If, $m' > E_i$, take $Q_i^* = \sum_{j=1}^{m'-1} \bar{F}_{ij}$ and stop,

otherwise go to Step (2).

After calculating the lot-size the next important step is to determine the reorder point for desired service level. This is described in next section.

3.8.3 Calculation of Reorder Point

An elaborate procedure has been used to determine the reorder level in case of the first group of medicines as they are important and tie up a considerable portion of the total investment in inventory. The procedure essentially consists of calculating the demand during lead time and its variance by using the lead time variance and forecast error. By scientifically adding adequate buffer to the average demand during lead time, desired service levels can be expected to be achieved.

The forecast error can be calculated at the same time when forecasts are being generated by using the following relationship:

$$\sigma_{if}^2 = \frac{\sum_{j=1}^{n_i'} (\bar{F}_{ij} - D_{ij})^2}{n_i'} \qquad (3.18)$$

where,

$\bar{F}_{ij}$ = estimated demand of item 'i' for period 'j',

D_{ij} = actual demand of item 'i' for period 'j',

$(\bar{F}_{ij} - D_{ij})$ = forecast error for period 'j',

σ_{if}^2 = variance associated with the next period forecast, $\bar{F}_{i,\ n'+1}$, of item 'i',

n'_i = number of time periods used in variance determination.

To be able to set reorder levels for various medicines for desired service levels, it is necessary to know the lead time frequency distribution. It has been found that in hospitals, in general, the stock items may be purchased from several vendors and the method of purchase from the same vendor may be by either bid or direct purchase order. These differences pose several problems in computing lead time parameters for the medicines. To overcome these restrictions total lead time has been broken up into three subelements and some assumptions have been made which are as follows:

Let the total lead time be denoted by T_i and further let T_{i1}, T_{i2} and T_{i3} be its three subelements, where,

T_{i1} = the time required to write and receive approval of the purchase requisition,

T_{i2} = the time required to write the bid quotation sheet plus, the time taken by vendor to consider this bid, and the time for the hospital staff to write the purchase order after accepting a bid,

T_{i3} = the elapsed time from issuance of the purchase order until the units are ready on the shelf for internal distribution.

We assume that 'T_{i1}' and 'T_{i2}' are the same for all medicines. Their values can be established from past data of all items and periodic review by sampling can be made to prevent any error due to abrupt changes in procedure. For nonbid items, time 'T_{i2}' will be equal to purchase order writing time, since the sublement bid time will be zero. We further assume that the three times are independent and, therefore the estimated mean value of the total lead time for an item can be taken as sum of the sampled mean values of T_{i1}, T_{i2} and T_{i3}. Denoting averages with a bar we have,

$$\bar{T}_i = \bar{T}_{i1} + \bar{T}_{i2} + \bar{T}_{i3} \tag{3.19}$$

and from the theorem of linear combinations applied to sample values,

$$\sigma^2_{T_i} = \sigma^2_{T_{i1}} + \sigma^2_{T_{i2}} + \sigma^2_{T_{i3}} \tag{3.20}$$

Let the demand during the lead time of an item be denoted by X_i, the mean of the sample distribution by $\bar{X}_i$, and the variance by $\sigma^2_{x_i}$. Then for the i-th item

$$\bar{X}_i = \frac{Q_i^* \bar{T}_i}{m'_i}, \quad \text{and} \tag{3.21}$$

$$\sigma^2_{x_i} = \sigma^2_{if} \bar{T}_i^2 + \sigma^2_{T_i} \left(\frac{Q_i^*}{m'_i} \right)^2 + \sigma^2_{if} \sigma^2_{T_i} \tag{3.22}$$

where Q_i^* is the lot size that will meet the requirements of m' periods ahead from now.

Since both the demand distribution and lead time distribution are unimodel, it can be safely assumed that the product of the two is also unimodal. The reorder level can then be computed using Camp and Meidel's extension of Tchebycheff's inequality which states:

$$P(X_i > \bar{X}_i + Z_i \sigma_{x_i}) \leq \frac{1}{4.5 Z_i^2} \tag{3.23}$$

where,

$P(X_i > \bar{X}_i + Z_i \sigma_{x_i})$ = probability of stockout

= (1-service level 'p'_{io})

$\bar{X}_i + Z_i \sigma_{x_i}$ = order point providing a probability of 'p_{io}' of not having a stockout during replenishment,

p_{io} = service level = probability of not being out of stock.

The inequality can be rewritten as,

$$Z_i \geq \sqrt{\frac{1}{4.5(1-p_{io})}} , \qquad (3.24)$$

and,

$$R_{io} = \bar{X}_i + Z_i \sigma_{x_i} . \qquad (3.25)$$

where,

R_{io} = reorder point for medicine 'i' .

Thus for a prescribed service level, 'p_{io}', Z_i can be computed readily from relation (3.24) which upon substitution in equation (3.25) gives the reorder point R_{io}. This completes the determination of all control parameters for 'Class-I' medicines.

3.9 Control Procedure for 'Class-II' Medicines

The second group is made up of those items which have a low annual rupee usage but are otherwise important. Due to this reason the same forecasting and lot sizing procedures have been used for this group as were used for the 'Class I' medicines. However, the procedure for calculating the reorder point for 'Class I' items is not economical for this class as the advantages gained from tighter control may be offset by the increased cost of data collection.

In order to reduce the data collection cost, the lead time estimate can be computed in the same manner as for 'Class I' items but for the class rather than individual medicines. This value can then be applied to each medicine in the group.

To find the lead time estimate, a large number of latest orders should be selected and if the number of observations is sufficiently large, their distribution can be assumed normal on the basis of the central limit theorem. That value of lead time can be taken for all items which is large enough to include, say h-percent, of the lead times of the items in 'Class-II'. This value can be readily estimated using normal distribution tables. The mean and

variance of the demand during lead time distribution for an item 'i' are given by the relationships,

$$\bar{X}_i = \left(\frac{Q_i^*}{m_i'}\right) T_{i(h\text{-percent})} \; ; \quad \text{and} \tag{3.26}$$

$$\sigma^2_{x_i} = \sigma^2_{if} \, T^2_{i(h\text{-percent})} \tag{3.27}$$

The order point can be found as before using Camp and Meidel's extension of Tchebycheff's inequality. As these items are important and their annual rupee usage is lower, we can opt for higher service levels.

3.10 Control Procedure for Class III Items:

These items have low overall importance and hence the usage of adaptive control limits does not seem to be economical as it involves considerable data collection effort. Therefore, fixed limit controls have been used for these medicines. No forecasting is needed on a period by period basis and the lot sizes have been computed using the classical EOQ formula which states that for an item 'i',

$$Q_i^* = \sqrt{\frac{(2 \, A_i \, C_{is}) \times 100}{P_i \, p_{ic}}} \tag{3.28}$$

where,

Q_i^* = optimal lot size,

A_i = annual demand in units,

C_{is} = set up cost per order

P_i = price per unit, and

p_{ic} = carrying charges expressed as percentage of 'P_i'.

This lot size is, of course, checked against the expiry time of the medicine.

In calculating the reorder point, instead of concentrating on accurate lead time estimates, it is more economical to provide larger buffers. The buffer is provided for a period 'T_{is}' and is obtained by adding this period to the 'h-percent' confidence limit lead time value as follows:

$$R_{io} = \left(\frac{Q_i^*}{m_i'}\right) T_{i(\text{h-percent})} + \left(\frac{Q_i^*}{m_i'}\right) T_{is}$$

or

$$R_{io} = \frac{Q_i^*}{m'_i} \left[T_{i(\text{h-percent})} + T_{is}\right] \qquad (3.29)$$

where 'T_{is}' is the fixed time factor whose value is assigned by the management. A greater value of 'T_{is}' is used if the lead time for the medicine is long and variable and if a stock-out of the medicine is particularly undesirable

from some other considerations.

3.11 Summary

In this chapter an inventory classification scheme based on fuzzy clustering concepts has been developed. Further a detailed account of the inventory control procedures to be used for various classes of medicines has been presented. The proposed inventory classification and control procedures have been tested using a case study on medicine inventory. The fuzzy inventory classification scheme is compared with the classical ABC analysis. The data for the case study has been collected from the Health Centre, Indian Institute of Technology, Kanpur. The results are presented in Chapter IV.

CHAPTER IV

A CASE STUDY : RESULTS, DISCUSSIONS AND THE SCOPE FOR FURTHER WORK

In order to validate the methodology developed during the course of this work, the medicine inventory control problems of the Health Center, Indian Institute of Technology, Kanpur were studied. The discussion on hospital medicine inventory control problems presented in the previous chapters was also to an extent motivated from the observations made there.

The currently used medicine inventory control procedure at IIT Kanpur Health Center is the same as described in Section (3.6). The specific problems encountered while designing a medicine inventory control system have been discussed in Section (3.3).

Data has been collected for 119 medicines which represent a cross-section of the medicine inventory at IIT Kanpur Health Center. These medicines have an annual usage of approximately Rs. 4,32,000. Even assuming that the inventory set-up and carrying costs add up to 10 per cent of the inventory investment, it is hoped that through

proper control procedures a saving of upto 25 percent, that is Rs. 10,000, can be realised annually on this small cross-section of the medicine inventory. The second important advantage will be the attainment of higher service levels which are of utmost important to service organisations like hospitals.

In the following sections results based on this case study will be presented. First a traditional ABC analysis has been performed which is followed by FICS (Fuzzy Inventory Classification Scheme) analysis. The results obtained from both the analyses have been compared. A computer program has been developed in FORTRAN IV to compute the parameters required for implementing the designed control system for various classes of medicines. The details regarding this program and other programs developed during this work are given in Appendix I.

4.1 ABC Classification

The procedure to perform annual rupee usage based ABC classification, given in Section (3.4.1), has been computerised. The listing of the computer program is given in Appendix II. The results of this analysis are as follows:

1. All the 119 medicines, taken together, have an annual usage of Rs. 4,32,237/ -.

2. The first 20 percent of the medicines, i.e. 23 medicines belonging to the A-class have an annual rupee usage of Rs. 307,520 which is 71.15 percent of the total.

3. The middle 30 percent of the medicines, i.e., 35 medicines belonging to the B-class have an annual rupee usage of Rs. 1,03,735 which is 24 percent of the total.

4. The last 50 percent of the medicines,i.e., 61 medicines belonging to the C-class have an annual rupee usage of Rs. 20,982 which is 4.85 percent of the total. The class to which a particular medicine belongs is given in Table 4.1. Figure 5 depicts the ABC curve based on this analysis.

4.2 Fuzzy Inventory Classification

The same set of medicines has been subjected to FICS analysis. A questionnaire given in Figure 6 has been used to collect the information about the various feature vector components. The first step in FICS is choosing the initial centroids.

4.2.1 Initial Centroids

Various sets of initial centroids have been generated by using the Ball and Hall approach described in Section (3.5.4). Figure 7 shows the variation in the number of centroids obtained with respect to the inter-centroidal distance. It indicates that as the requifed separation increases, the

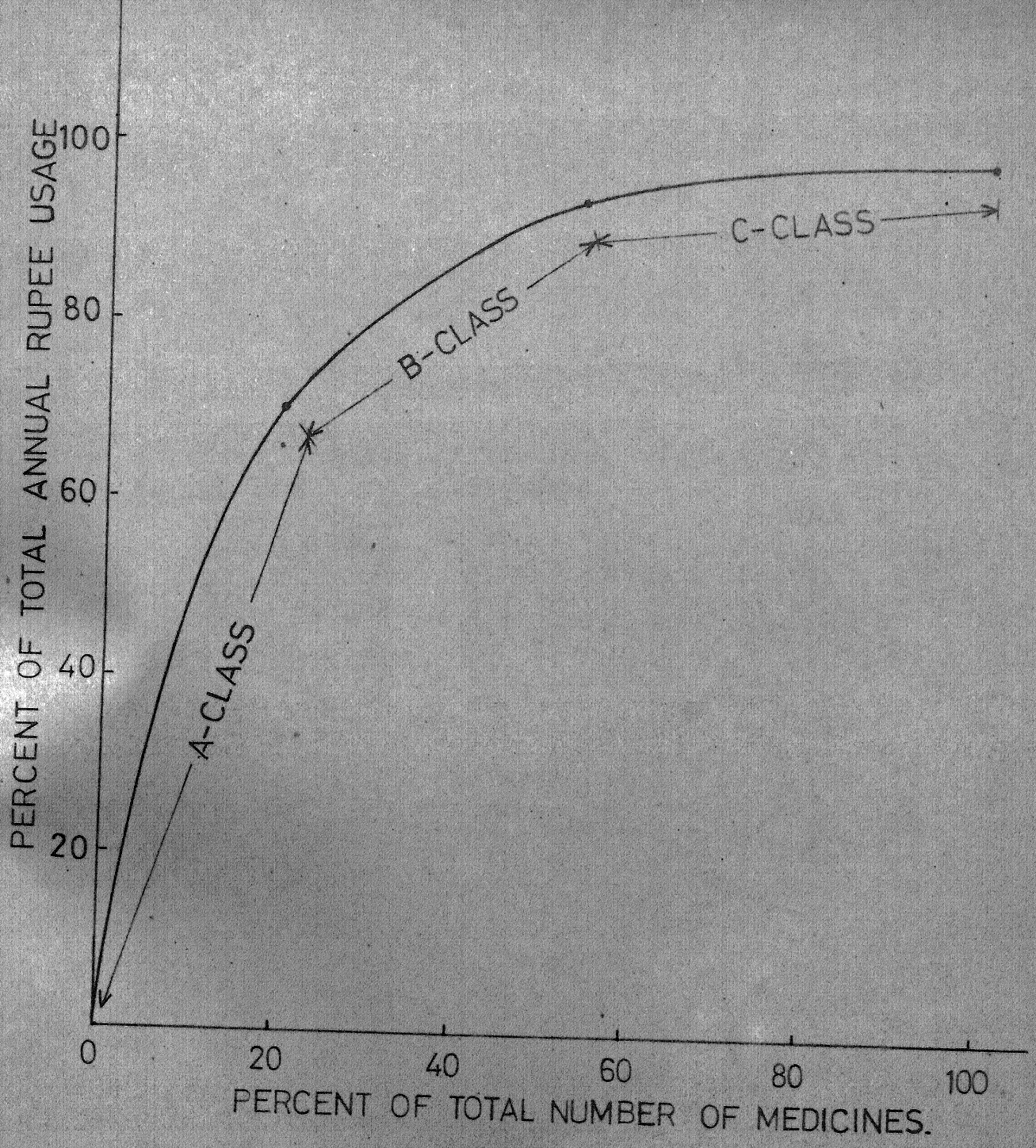


FIGURE-5

ANNUAL RUPEE USAGE CURVE OF SAMPLE OF 119 MEDICINES STOCKED AT IIT KANPUR HEALTH CENTRE.

Table 4.1: Membership grades of medicines in the three clusters.

Sl. No.	Name of the Medicine	Membership Grade in			'FICS' Class	Classical ABC Class
		Class I	Class II	Class III		
1.	ACTIDIL TABLET	0.20731311	0.72994550	0.06274141	II	B
2.	ADEXOLIN LIQUID	0.72288808	0.26916566	0.00794626	I	C
3.	ADELPHAN TABLET	0.06392029	0.19654927	0.73953044	III	C
4.	ALDOMET AND METHYLTOPA TABLET	0.00756762	0.99154652	0.00088589	II	B
5.	ALGESIN, BUTARIN AND HEMALIN TABLET	0.99298444	0.00683324	0.00018233	I	B
6.	AMINOPHYLLIN TABLET	0.00376297	0.99483418	0.00140285	II	C
7.	ANGISED TABLET	0.24460191	0.49348984	0.26190827	II	C
8.	ANALGIN TABLET	0.87176260	0.11322380	0.01501362	I	A
9.	ANTEPAR ELIXIR/ENTACYL ELIXIR	0.00258983	0.00082996	0.99658022	III	C
10.	ASCABIOL EMULSION	0.21219139	0.00683324	0.73975088	III	A
11.	ASPIRIN AND MICROPYRIN PLAIN TABLET	0.00220781	0.00085476	0.99693744	III	C
12.	BARALGAN TABLET	0.70242982	0.04600994	0.25156026	I	A
13.	BATNOVIT-C SKIN OINTMENT	0.96785035	0.03175554	0.00039413	I	A
14.	B-COMPLEX TABLET	0.99397171	0.00278708	0.00324122	I	A
15.	BENTELAN TABLET	0.13183954	0.03680823	0.83135224	III	C
16.	BUTAZOLIDIN TABLET	0.36947704	0.62791812	0.00260485	II	B
17.	CALMPOSE TABLET	0.00109645	0.99881054	0.00009302	II	C
18.	CHLORAMPHENICOL-250 MG CAPSULE	0.98247965	0.01640083	0.00111954	I	A
19.	CHLOROMYCETIN APPLICAPS	0.78027226	0.21659496	0.00313280	I	B
20.	DESULAN TABLET	0.00052008	0.99938682	0.00009603	II	C
21.	DIETHYL CARBENZYME	0.52233017	0.31467989	0.16298994	I	A
22.	DINABOL - 1 MG TABLET	0.00049907	0.99940048	0.00010045	II	C
23.	DINABOL - 5 MG TABLET	0.00052008	0.99938434	0.00009560	II	C
24.	DRAMAMINE TABLET	0.00000604	0.00000401	0.99998997	III	C
25.	DRISTAN TABLET	0.99866331	0.00066932	0.00066739	I	A
26.	DUOGYNON TABLET	0.00052480	0.99938021	0.00009501	II	C
27.	EMYCIN AND ERYTHROCIN DRY SYRUP	0.71913685	0.26728639	0.01357678	I	A
28.	EMACYIN TABLET	0.98597819	0.00658480	0.00743703	I	A
29.	ENTACYL AND PIPPRAZINE TABS.	0.02032849	0.08141421	0.89825731	III	C
30.	ENTROSTREP AND STREPTO PARAXIN CAPSULE	0.99827413	0.00085315	0.00087274	I	A

contd

1.	2	3	4	5	6	7
66.	N.F.T. TABLET	0.00000592	0.00000390	0.99999020	III	C
67.	PELONIN - 250 MG TABLET	0.00000611	0.00000407	0.99998982	III	C
68.	PENICILIN -65 MG TABLET	0.99986000	0.00011193	0.00002808	I	A
69.	PENICILIN - 130 MG TABLET	0.99504268	0.00344803	0.00150930	I	A
70.	PENZYME/UNIENZYME TABLET	0.96388423	0.01604895	0.02006683	I	B
71.	PHENOBARBITILON TABLET	0.00000600	0.00000397	0.99999006	III	C
72.	POLLIO VACCINE DROP	0.01155633	0.98673066	0.00171301	II	C
73.	PREPALIN -A TABLET	0.04582725	0.00950255	0.94467022	III	C
74.	PROLOID TABLET	0.00000609	0.00000406	0.99998987	III	C
75.	PYRIDIUM TABLET	0.07061600	0.87068459	0.05869942	II	C
76.	REDOXON - 500 MG TABLET	0.04287301	0.00310403	0.95402297	III	A
77.	SEDONAL TABLET	0.03837230	0.14576141	0.81586631	III	B
78.	SIQUIL 100 MG TABLET	0.00691362	0.99219245	0.00089397	II	C
79.	SISTOMETRIL TABLET	0.00080775	0.99909310	0.00009917	II	B
80.	SONERYL TABLET	0.06492905	0.01271800	0.92235298	III	C
81.	STEMETIL TABLET	0.00165310	0.99710646	0.00124046	II	C
82.	STYPTOVIL TABLET	0.96536821	0.02902992	0.00560188	I	B
83.	SULPHACETAMIDE EYE DROPS	0.88785820	0.05552836	0.05661345	I	B
84.	SULPHA TABLET	0.73724457	0.25908633	0.00366912	I	B
85.	TENDRIL TABLET	0.82684615	0.15689217	0.01626170	I	B
86.	TERRAMYCIN EAR DROP	0.84890936	0.11258758	0.03850309	I	B
87.	TERRAMYCIN EYE OINTMENT	0.59319009	0.40447212	0.00233779	I	B
88.	TETRACYCLINE -250 MG OINTMENT	0.93605726	0.05055608	0.01338668	I	A
89.	TETRACYCLINE -500 MG CAPSULE	0.94019400	0.05166869	0.00813732	I	A
90.	TINEAFOX POWDER	0.00000592	0.00000390	0.99999020	III	C
91.	UROLUCOSIL TABLET	0.12433880	0.81828237	0.05737886	II	B
92.	VASPAREX TABLET	0.00969130	0.01473401	0.97557470	III	C
93.	YEAST TABLET	0.14707257	0.00695162	0.84597583	III	C
94.	ADERNALIN INJECTION	0.07592174	0.91699199	0.00708631	II	C
95.	ANTI-SERRUM INJECTION	0.01136557	0.98690748	0.00172696	II	C
96.	AMINOPHYLLINE INJECTION	0.05282487	0.86277483	0.08400320	II	C
97.	ATS INJECTION	0.72455820	0.27298789	0.00245392	I	B
98.	ATROPIL INJECTION	0.00562350	0.99266914	0.00170739	II	C
99.	AVIL INJECTION	0.07214777	0.87296848	0.05488375	II	C
100.	BARALGAN INJECTION	0.02993215	0.93298691	0.03708096	II	C
101.	CALCIUM SANDOZ WITH VITAMIN C INJECTION	0.22748170	0.76669250	0.00582581	II	C
102.	CORAMIN INJECTION	0.03278422	0.96329916	0.00391663	II	C
103.	DECADRON INJECTION	0.40124512	0.58817648	0.01057841	II	B
104.	DERIPHLLIN INJECTION	0.00366589	0.99544577	0.00088835	II	C

cont'd

1.	2	3	4	5	6	7
105.	DEXTROSE - 5 PC INJECTION	0.04815407	0.95024896	0.00159699	II	B
106.	DEXTROSE -25 PC INJECTION	0.02505041	0.97365417	0.00129545	II	C
107.	DEXTROSE SALINE INJECTION	0.07954109	0.91881287	0.00164608	II	B
108.	MORPHINE INJECTION	0.01136759	0.98690565	0.00172678	II	C
109.	NOVOCAIN INJECTION	0.00174954	0.99583755	0.00241294	II	C
110.	PETHADINE INJECTION	0.01136759	0.98690565	0.00172678	II	C
111.	PROLUTON INJECTION	0.00184427	0.99584129	0.00231445	II	C
112.	SIQUIL INJECTION	0.00633952	0.99301407	0.00064644	II	C
113.	STEMETIL INJECTION	0.02679244	0.94730292	0.02590466	II	C
114.	STREPTOMYCIN INJECTION	0.98853184	0.00580101	0.00566716	I	B
115.	STREPTOPENITILLIN INJECTION	0.99305999	0.00580442	0.00113562	I	B
116.	STYPTCHROME INJECTION	0.23277889	0.71997388	0.04724724	II	C
117.	TETIVACC/PTAP INJECTION	0.24854733	0.74701682	0.00443587	II	C
118.	TERRAMYCIN INJECTION	0.18922514	0.78311973	0.02765515	II	C
119.	WATER FOR INJECTION	0.56490293	0.05060807	0.38448900	I	A

Figure 6: Questionnaire used for data collection.

Questionnaire

1. Name of the Item: _____
2. Cost per unit: _____
3. Annual consumption: _____
4. Annual rupee usage: _____
5. Expiry time in months: _____
6. Maximum percent quantity discount available: _____
7. Set up cost per order: _____
8. Carrying cost/item/month: _____
9. Item type: V. Important Life Saving ☐ Important ☐ Ordinary ☐
10. Substitution: Never Possible ☐ Sometimes ☐ Always ☐
11. Market Availability: Difficult ☐ Sometimes not available ☐ Always ☐
12. Shortage history: Frequently short ☐ Sometimes short ☐ No stock outs ☐
13. Storing method: Special care ☐ Some care ☐ No care ☐
14. Storage space Occupies too much ☐ Average ☐ Low ☐

Fig. 6: Form for Lead Time and Demand Data Collection.

	LEAD TIME		
No.	T1	T2	T3
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			

DEMAND HISTORY			
MONTH	YEAR	YEAR	YEAR
JAN			
FEB			
MAR			
APR			
MAY			
JUN			
JUL			
AUG			
SEP			
OCT			
NOV			
DEC			
TOTAL			

T1 = Time when shortage noticed to time when purchase requisition accepted.

T2 = Time of acceptance of purchase requisition to time when order mailed.

T3 = Time from mailing of orders to items ready on shelf for distribution.

T = T1 + T2 + T3

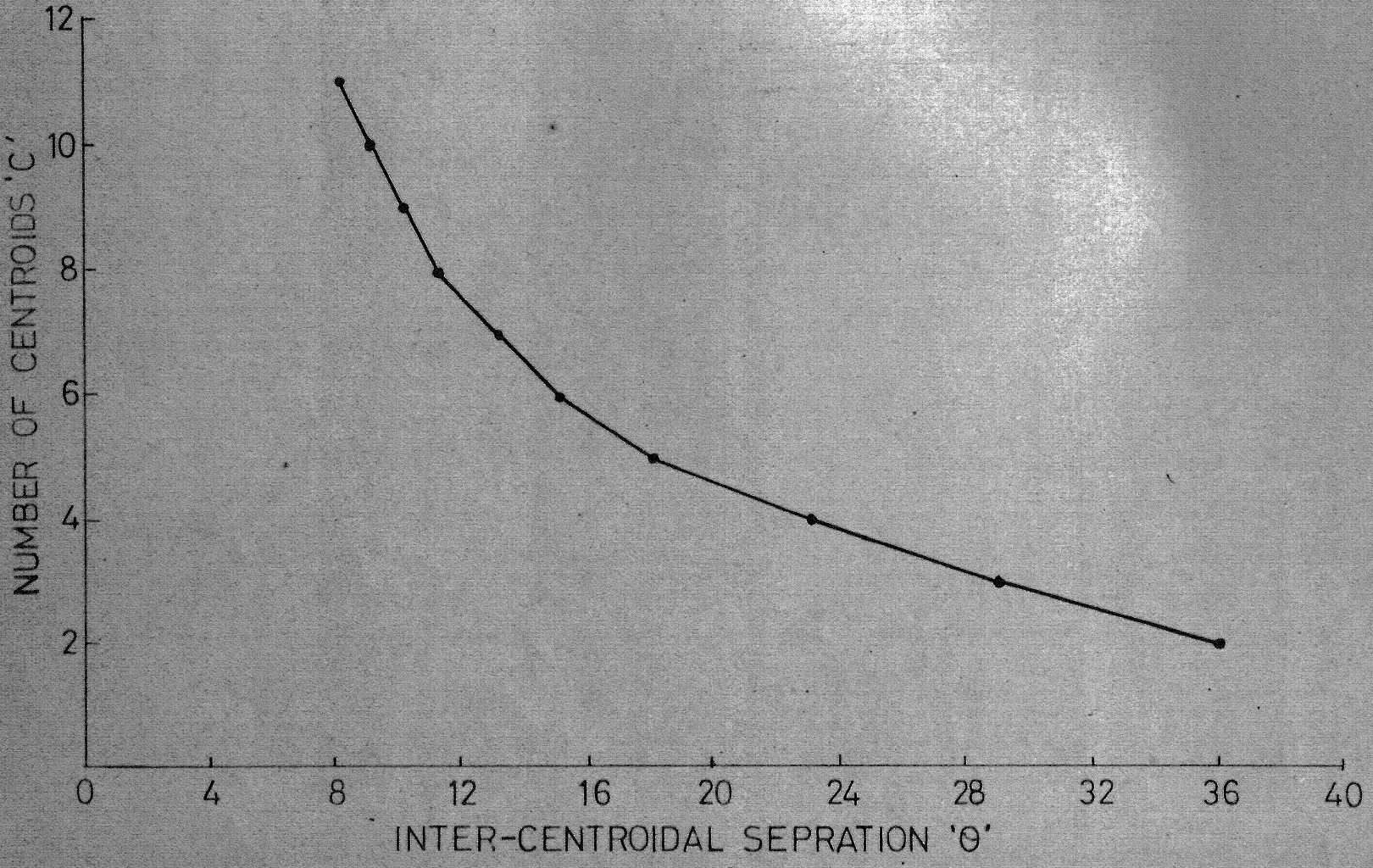

NUMBER OF CENTROIDS 'C'
12
10
8
6
4
2
0
4
8
12
16
20
24
28
32
36
40
INTER-CENTROIDAL SEPRATION 'θ'

FIG. 7

number of centroids generated decreases. These initial centroids have been taken as input to the Fuzzy ISODATA clustering algorithm.

4.2.2 Partitioning by Fuzzy ISODATA

For a given set of 'c' initial centroids, several fuzzy partitions have been generated by varying the values of the exponent 'm'. Equal weights have been assigned to all the feature vector components during this process. The experiments have been conducted for c = 2,3 and 4 and both the measures of partition quality, H_c and F_c, have been evaluated for each fuzzy c-partition. Table 4.2 lists values of F_c, $1-F_c$ and H_c for the partitions obtained for various values of 'c' and 'm'. Figures (8), (9) and (10) depict the variation in F_c and H_c with respect to 'm' for c = 2,3, and 4 respectively. It can be seen from Table 4.2 that with 'c' fixed, $H_c \longrightarrow 0$ and $F_c \longrightarrow 1$ as 'm' $\longrightarrow 1$. This is expected since $J_m \longrightarrow J_1$, and the partitioning is optimal for J_1 only when it lies in M_c, the hard partition space associated with X [22]. Thus as the value of 'm' increases the fuzziness of the c-partitions of X obtained with Fuzzy ISODATA also increases.

Now scanning Table 4.2 for minimum H_c, which is a better measure of quality [22], we observe that the entropy is least for c = 3 for every value of 'm'. From this we

Table 4.2: Quality of Clusters for Various 'm' and 'c' Values.
$(1-F_c)$ = Lower Bound on H_c

Sl. No.	m	C = 2			C = 3			C = 4		
		F_c	$(1-F_c)$	H_c	F_c	$(1-F_c)$	H_c	F_c	$(1-F_c)$	H_c
1.	1.10	0.95861	0.04139	0.07342	0.95923	0.04077	0.07290	0.94643	0.05357	0.09211
2.	1.15	0.92133	0.07867	0.14615	0.92176	0.07824	0.13810	0.90808	0.09192	0.16263
3.	1.20	0.88114	0.11886	0.21763	0.88240	0.11760	0.20745	0.86766	0.13234	0.23855
4.	1.25	0.84588	0.15412	0.28003	0.84692	0.15308	0.27113	0.82240	0.17760	0.32305
5.	1.30	0.81218	0.18782	0.34201	0.81235	0.18765	0.33312	0.78166	0.21834	0.40201
6.	1.35	0.77765	0.22235	0.40194	0.77896	0.22104	0.39256	0.74460	0.25540	0.47417
7.	1.40	0.74003	0.25997	0.45367	0.74619	0.25381	0.44985	0.70941	0.29059	0.54247
8.	1.45	0.71440	0.28560	0.51169	0.71467	0.28533	0.50382	0.67666	0.32334	0.60588
9.	1.50	0.68031	0.31967	0.56240	0.68462	0.31538	0.55413	0.64464	0.35536	0.66688
10.	1.55	0.64987	0.35013	0.61488	0.65308	0.34692	0.60627	0.61865	0.38135	0.71604
11.	1.60	0.62936	0.37064	0.64986	0.63094	0.36906	0.64201	0.59093	0.40907	0.76823
12.	1.65	0.60550	0.39450	0.68533	0.60774	0.39226	0.67945	0.56601	0.43399	0.81502
13.	1.70	0.58619	0.41381	0.72011	0.58625	0.41375	0.71347	0.54346	0.45654	0.85723
14.	1.75	0.56312	0.43688	0.75346	0.56679	0.43321	0.74408	0.52057	0.47943	0.89921
15.	1.80	0.54201	0.45799	0.78254	0.54924	0.45076	0.77161	0.50042	0.49958	0.93613
16.	1.85	0.52668	0.47332	0.81122	0.53339	0.46661	0.79644	0.47920	0.52080	0.97408
17.	1.90	0.51859	0.48141	0.83967	0.51901	0.48099	0.81892	0.45887	0.54113	1.00996
18.	1.95	0.49923	0.50077	0.86215	0.50499	0.49501	0.84054	0.42627	0.57373	1.06356
19.	2.00	0.49162	0.50838	0.88311	0.49297	0.50703	0.85927	0.41470	0.58530	1.08507
20.	2.05	0.47856	0.52144	0.89756	0.48231	0.51769	0.87578	0.40426	0.59574	1.10462
21.	2.10	0.46914	0.53086	0.91379	0.47164	0.52836	0.88521	0.39453	0.60547	1.12290
22.	2.15	0.45895	0.54105	0.92188	0.46198	0.53802	0.90706	0.38791	0.61209	1.13513
23.	2.20	0.44777	0.55223	0.93153	0.45231	0.54769	0.92162	0.37986	0.62014	1.15026

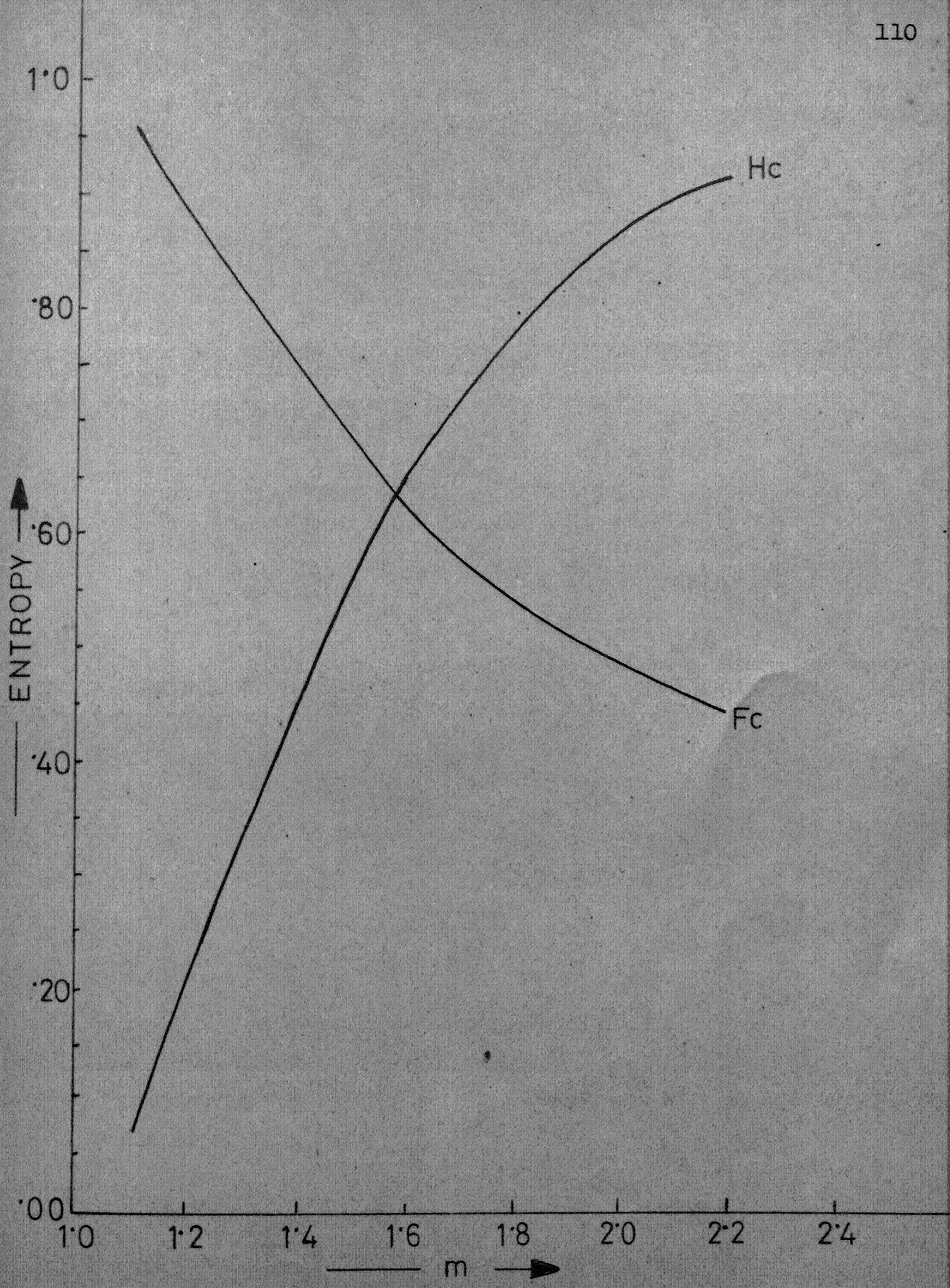


FIG. 8

VARIATION IN Hc AND Fc WITH EXPONENT 'm' FOR c = 2

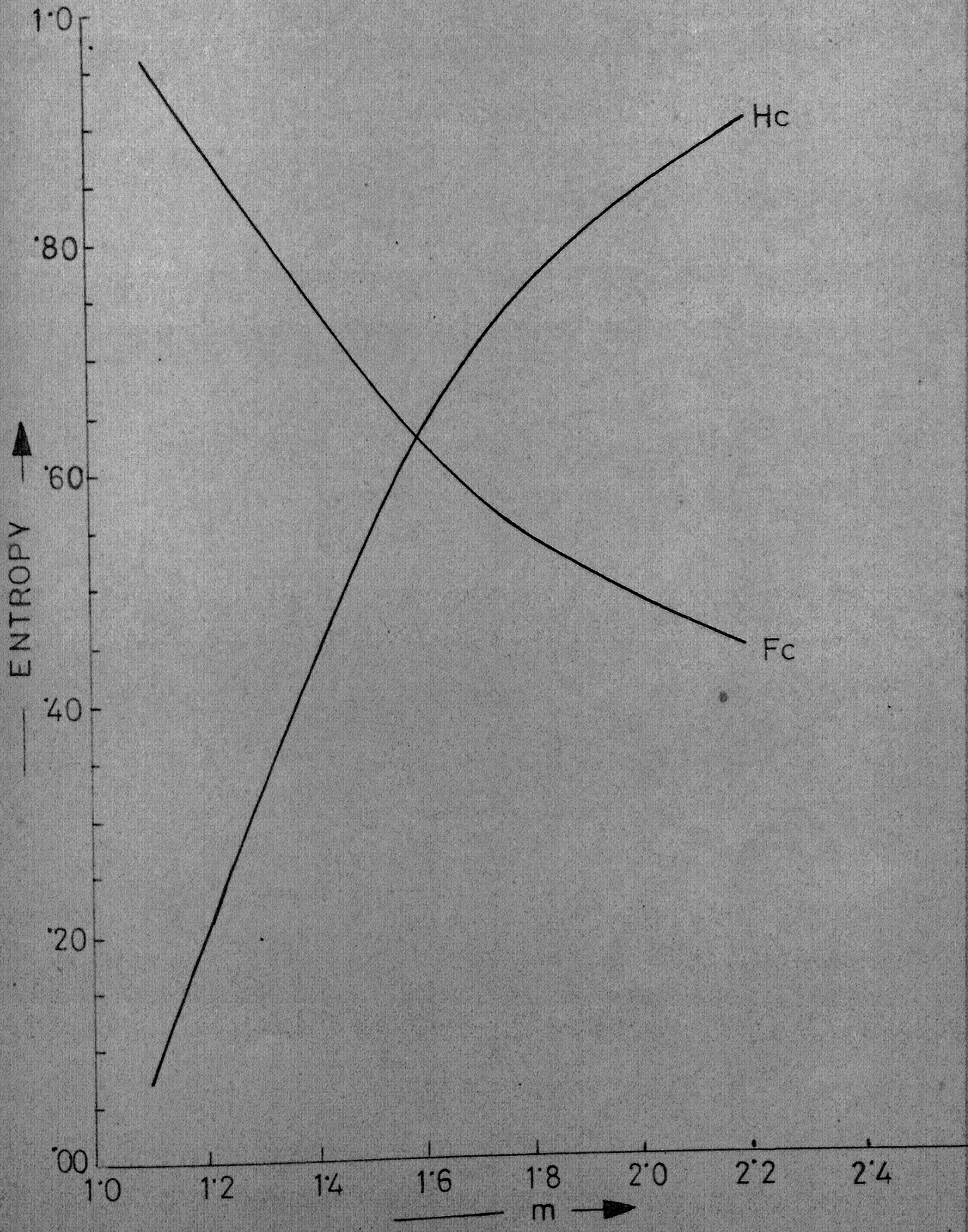


FIG.9
VARIATION IN Hc AND Fc WITH EXPONENT 'm' FOR c=3.

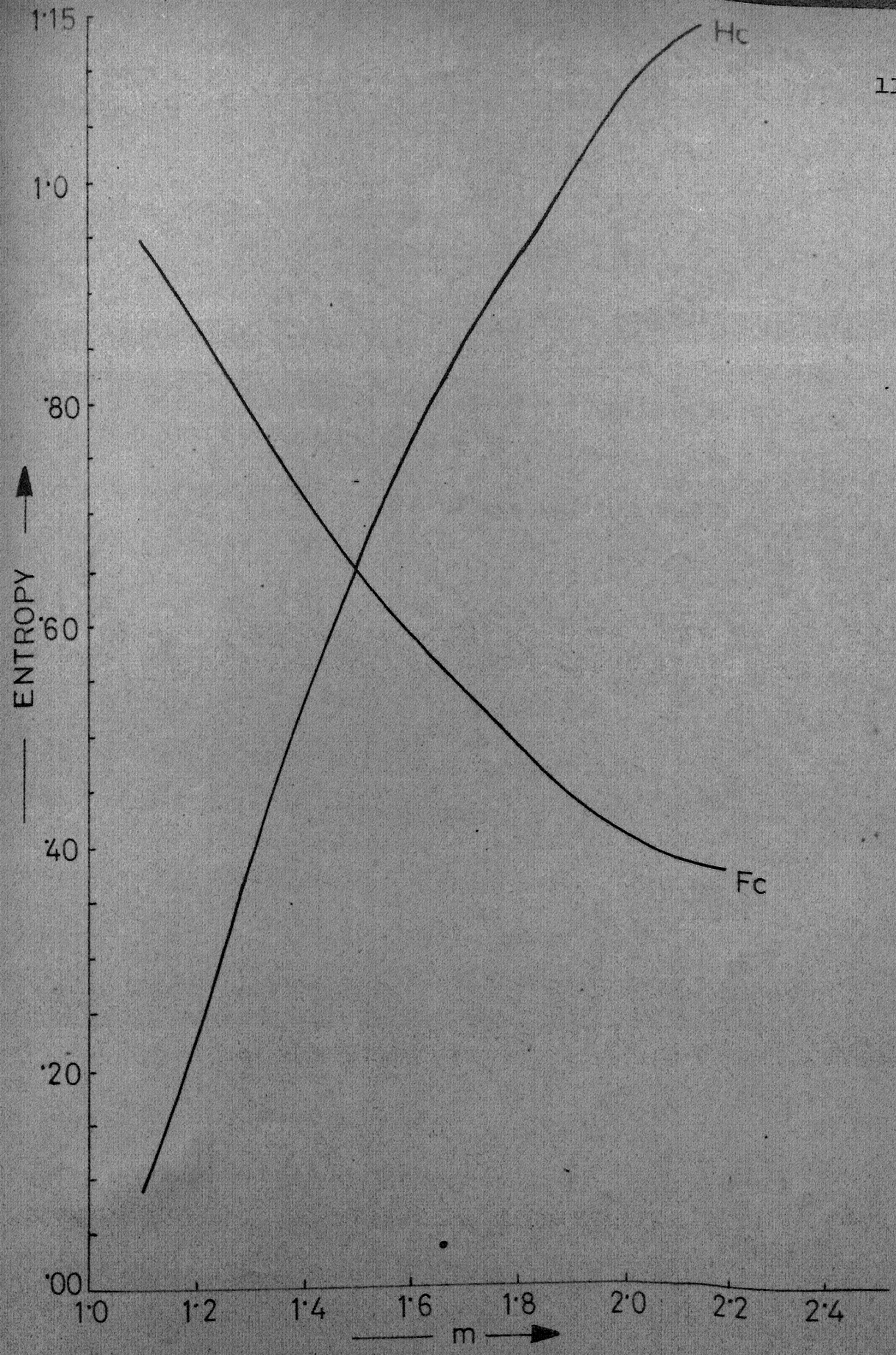


FIG. 10
VARIATION IN Hc AND Fc WITH EXPONENT 'm' FOR c=4

infer that the most plausible choice for primary substructure in X is 3 subclasses. As no theoretical result suggesting an optimal choice of 'm' has been developed to date [22], further discussion is based on, $c = 3$ and $m = 1.25$, which appear to be satisfactory for our purpose.

The initial centroids for starting the Fuzzy ISODATA process are:

$$\underset{\sim}{v}_1 = \{0.75, 0.93, 1, 1, 1, 0.5, 0.5, 1\}$$
$$\underset{\sim}{v}_2 = \{0.12, 0.27, 0.45, 0.36, 0.41, 0.36, 0.44, 0.52\}$$
$$\underset{\sim}{v}_3 = \{0.01, 0.75, 0, 0, 0, 0, 0, 1\}$$

The final centroids which have been obtained at the end of the classification process are:

$$\hat{\underset{\sim}{v}}_1 = \{0.2842, 0.6196, 0.4768, 0.2433, 0.4619, 0.3572, 0.5032, 0.8790\}$$
$$\hat{\underset{\sim}{v}}_2 = \{0.0412, 0.1162, 0.6850, 0.6569, 0.6098, 0.5595, 0.5997, 0.4379\}$$
$$\hat{\underset{\sim}{v}}_3 = \{0.0499, 0.1139, 0.0721, 0.0545, 0.0613, 0.0589, 0.1449, 0.2405\}$$

These centroids essentially represent the three subclass present in the data set. The first centroid has distinctly higher annual rupee usage and expiry time components (i.e.,

first two components of the feature vector) as compared to the other two centroids. Moreover, the values of its other components are also substantial. This clearly indicates that the first centroid represents the group of those medicines which have a higher annual rupee usage and shorter expiry duration as compared to the medicines belonging to the other two classes. The other components of this centroid also show its considerable overall importance. Therefore, we conclude that the medicines belonging to 'Class-I', represented by this centroid, are important from all considerations and should be controlled by the best procedure.

The second centroid has low annual rupee usage and expiry time components but its other components advocate thatsubstantial importance be assigned to the group it represents. In fact, this centroid essentially represents the group of those medicines which have lower annual rupee usage and larger expiry period but are otherwise quite important. A good number of injections belong to this group. However, there are some medicines in this group which have low expiry time. This is an example of misclassification. But such medicines can be detected as they have a lower membership grade in this group as compared to the other medicines belonging to it. Two such examples are CALCIUM SANDOZ INJECTION and DECADRON INJECTION. Anyway as we are making a

check during the lot sizing calculations to see that an order is not placed for a period greater than the expiry time of the medicine, the errors due to this misclassification will not be serious. As these medicines are important and have low annual rupee usage and high expiry time , we can opt for higher service levels for them without sacrificing much in terms of costs.

The third centroid has distinctly low values of all the feature vector components and it obviously represents the medicines which are unimportant from all considerations. A relatively loose control procedure can therefore be used for this group of medicines.

We would like to emphasise here that any conversion of the fuzzy partitions to hard partitions should not be made at once by using some rule like the 'maximum membership grade rule'. Instead we should leave the solution fuzzy to identify the individuals in the data whose characteristics are shared by competing subclasses. When this has been thoughtfully considered, for all practical purposes the partitions may be converted to the hard c-partitions by using a suitable rule. This will greatly help in avoiding any misclassifications.

We have converted the three fuzzy partitions in to hard partitions by using the maximum membership grade rule and the resulting classification has been compared with partitions obtained from the classical ABC analysis.

4.3 ABC Analysis v/s FICS

Table 4.2 shows the membership grades of the medicines in thethree classes which have been identified by FICS. It also shows the class to which they belong to after the fuzzy partitions have been converted into hard partitions by applying the 'maximum membership grade rule' and the class to which they belong in a traditional 20-30-50 ABC classification. Whereas in an ABC classification system the most important, important and unimportant items belong to the A, B and C classes respectively, in FICS the corresponding classes will be 'Class I', 'Class II' and 'Class III'. Table 4.3 shows the percentage of medicines falling in each group of the two classifications.

Table 4.3: Comparison of ABC Analysis with FICS

Classification system	Percentage of items in Class		
	I or A	II or B	III or C
ABC	20	30	50
FICS	32.77	41.17	26.06

The details regarding the changes in the membership of various items in the three classes are as follows:

1. Out of the total of 119 medicines, no change has taken place in the classes of 61 medicines.

2. No transfers have taken place from 'Class A' to 'Class II' and only two transfers have been recorded from 'Class A' to 'Class III', This implies that out of the total 23 medicines belonging to 'Class A', 21 have retained their memberships in the most important class of FICS. The two medicines transferred are ASCABIOL EMULSION and REDOXON - 500 mg tablets. An examination of their feature vectors will reveal that they are really unimportant from considerations other than the annual rupee usage. The feature vectors are

Ascabiol Emulsion = $\{0.23, 0, 0, 0.5, 0, 0,5, 0, 1\}$

Redoxon-500 mg Tablet = $\{0.29, 0, 0, 0, 0, 0, 0, 1\}$

Hence these changes are quite justified and are according to the philosophy of selective inventory management.

3. Sixteen medicines from 'Class B' have been promoted to 'Class I'. These changes are also satisfactory since their feature vectors justify their claim for being controlled by better procedure. For example, Penzyme and Unienzyme Tablets which belongs to this class have a

feature vector,

$$\{.053, 1, 0, 0, .5, .5, .5, 1\}$$

4. Four medicines have been downgraded to 'Class III' from 'Class B'. Their overall low importance is evident from their feature vectors. The feature vectors are,

LIV - 52 Tablet = $\{0.10, 0, 0, 0, 0, 0, 0, 1\}$

LIV - 52 Drops = $\{0.09, 0, 0, 0, 0, 0, 0, 1\}$

Mejoral Tablet = $\{0.10, 0, 0, 0, 0, 0, 0, 1\}$

Sedonal Tablet = $\{0.05, 0, 0, 0,5, 0,5, 0, 0.5, 0\}$

5. Two medicines for which the feature vectors are given below, have been promoted from 'Class C' to 'Class I'.

Adexolin Liquid = $\{.018, 1, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5\}$

Glaxena Tablet = $\{.023, 0.928, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5\}$

The feature vectors of these medicines also favour this change.

6. Thirty four medicines have been promoted from 'Class C' to 'Class II'. A good number of them are injections which have low annual rupee usage but are important from other considerations. This is the peculiar characteristic of 'Class II', and therefore, this change is also rational.

From the observations above we conclude that the FICS has been very successful in recognising the underlying importance groups in the medicine inventory and the changes brought about by it in the tranditional ABC classification are justified. It has correctly classified the medicines on the basis of their overall importance in order to subject them to proper control procedures.

4.4 Integrated Forecasting and Lot-Sizing

A computer program has been developed which forecasts the demands and computes the lot sizes and reorder levels for all the three classes of medicines as per the procedures described in Sections (3.8, 3.9, 3.10). It also points out related information on the investment required for each item and expected time for placing the next order. A sample output has been shown in Table 4.4. The program works on independent ordering policy for all medicines and whenever a replenishment order is to be placed, it needs to be run only for the required medicines.

4.5 Computational Experience

The greatest drawback of fuzzy clustering methods is a relatively large memory requirement. The core memory needed for FICS exceeded 16 K. However, with new and more powerful computer systems this should not remain a handicap.

Item No.	Name of the Item	Optimal order quantity (Q^*)	Reorder Level (R_o)	Unit Price in Rs.	Amount in Rs. to be invested	No. of periods covered	Next order to be placed in month
1.	ALGESIN, BUTARIN AND HEMALIN TABLETS	7076	3913	0.17	1202.92	6	7
2.	ANALGIN TABLET	16259	10568	0.17	2764.03	3	4
3.	BARALGAN TABLET	7392	4558	0.30	2217.60	3	4
4.	B-COMPLEX TABLET	27580	19548	0.10	2758.00	3	4
5.	BITNOVIT-C SKIN OINTMENT	667	279	3.50	2334.50	4	5
6.	BUTAZOLIDIN TABLET	7742	3039	0.18	1393.56	6	7
7.	CHLORAMPHENICOL - 250 MG CAPSULE	6748	5080	0.40	2699.20	2	3
8.	DIETHYLCARBENZYME	9422	6056	0.28	2638.16	3	4
9.	DRISTAN TABLET	13542	8316	0.15	2031.30	4	5
10.	EMYCIN DRY SYRUP	364	495*	6.00	2184.00	3	4
11.	EMYCIN TABLET	4904	4493	0.75	3678.00	3	4
12.	ENTROSTREP SUSPENSION	405	246	5.00	2025.00	4	5
13.	ENTROSTREP AND STREPTO PARAXIN CAPSULES	9092	7579	0.37	3364.04	4	5
14.	ENTROVIOFORM TABLET	11358	15931*	0.65	7382.70	2	3
15.	ESKAZINE TABLET	4226	1315	0.25	1056.50	9	10
16.	EXPETORANTS AND COUGH PREPARATIONS	303	449*	15.00	4545.00	1	2
17.	FLAGYL TABLET	3939	1945	0.52	2048.28	4	5
18.	FURAMIDE TABLET	5718	4864	0.47	2687.46	4	5
19.	FUROXONE SUSPENSION	473	150	4.50	2128.50	5	6
20.	FUROXONE TABLET	9597	11908*	0.19	1823.43	8	9
21.	HEXAVITAMIN TABLET	17874	5437	0.07	1251.81	7	8
22.	IRON PREPARATION (FERSOLATE/ DUMASULE)	19625	12533	0.15	2943.75	4	5
23.	LIV-52 DROPS	450	100	3.00	1350.00	5	6
24.	LIV-52 TABLET	18258	4383	0.08	1460.64	5	6

contd...

1	2	3	4	5	6	7	8
25.	MEJORAL TABLET	18267	4521	0.08	1461.36	5	6
26.	MULTIVITAMIN DROPS (BECADEX/VI-MAGNA)	1516	915	2.00	3032.00	3	4
27.	MEXAFORM TABLET	8391	7005	0.71	1426.47	6	7
28.	PENICILIN-65 MG TABLET (FENOCIN-65 MG)	12921	11544	0.20	2584.20	4	5
29.	PENICILIN -130 MG TABLET (FENOCIN-130 MG)	16282	16336*	0.39	6349.98	2	3
30.	POLLIO VACCINE DROP	91	40	1.00	91.00	3	4
31.	REDOXON-500 MG TABLET	9661	3836	0.25	2415.25	3	4
32.	ADERNALIN INJECTION	31	14	0.60	18.60	12	1
33.	AMINOPHYLLINE INJECTION	366	22	1.00	366.00	21	10
34.	A.T.S. INJECTION	448	99	3.00	1344.00	5	6
35.	AVIL INJECTION	817	110	1.00	817.00	9	10
36.	BARALGAN INJECTION	609	55	0.90	548.10	14	3
37.	CALCIUM SANDOZ WITH VITAMIN-C TNJECTION	385	44	1.80	693.00	11	12
38.	DEXTROSE - 5 PERCENT INJECTION	135	33	11.00	1485.00	5	6
39.	DEXTROSE SALINE INJECTION	135	33	11.00	1485.00	5	6
40.	WATER FOR INJECTION	12172	877	0.36	4381.92	1	2

For these items, $R_o > Q^*$, and therefore multiple - reorder points will be used. The orders will be placed when the stock level reaches 'R_o', '$R_o - Q^*$', '$R_o - 2Q^*$', etc.

The times taken by various programs on IBM 7044/1401 system are as follows:

(i) Classical ABC analysis - 32 seconds,

(ii) Initial centroids by Ball and Hall method - 46 seconds for all sets of centroids,

(iii) Fuzzy ISODATA classification for a single value of 'c' and 'm' - on average 60 seconds,

(iv) For lot size reorder point calculation - 30 seconds for 40 items.

Though a good amount of computer time will be spent in initially finding the number and types of classes present in inventory, this has to be done only when considerable changes in data profile have occurred. However, once the various subclasses have been established, the subsequent reclassification runs will not take much computer time.

4.6 Scope for Further Work

In the course of the development of the methodology presented in this dissertation, it was felt that further research needs to be undertaken in several directions. The potential areas of investigation and the possible extensions are:

1. The inventory control system is semi-computerised in its present form. The records are still maintained in a ledger and manual updating and checking is required to ensure the timely replenishment of medicines. As a further extension, the system can be fully computerised for automatic data processing. All records can be maintained on storage devices on computer and the transactions can be directly entered into the files for automatic updating. Whenever the stocks deplete below the reorder point, exception reports may be printed out.

2. It has been pointed out earlier in this thesis that the proposed inventory control system can be easily extended to suit the requirements of drug warehouses, medicine shops and retail stores. It will need only slight modification in the feature vectors. This type of research will help in further validation of the proposed methodology.

3. The applicability of more sophisticated techniques for forecasting the demands of a large number of items should be studied and if found suitable they can be incorporated in the proposed inventory control system.

4. Joint ordering policies may be tried for medicines. For better results, the dynamic lot sizing models accounting

for varying rates of return and prices and price breaks can be incorporated into the proposed inventory control system.

5. Extensive research needs to be undertaken in the theory of fuzzy sets. As we have pointed out earlier in this chapter, so far no theoretical results are known to optimally choose initial centroids and the exponent 'm' for fuzzy clustering. Further research regarding this will greatly reduce the computational burden and make the algorithms more effective.

6. More attention should be paid to the selection of features because contrary to one's intuition, adding more features does not always lead to better performance of classification algorithms [52]. We have presented the results only for the case when all the features are assigned equal weights. Further research should be undertaken to study the weightage sensitivity of the algorithm.

Lastly, the theory of fuzzy sets seems to have many potential applications in industrial engineering and attempts should therefore be made to use this approach in the areas where the situations are intrinsically fuzzy.

4.7 Summary

We have seen that the Fuzzy Inventory Classification Scheme is more general and compatible with the concept of Selective Inventory Management. In this sense, it is much more superior than the classical ABC analysis. Further, with the introduction of scientific inventory management in hospitals, significant improvements can be expected in the present cost structure and service level. The theory of fuzzy sets itself shall prove to be a powerful tool in analysing the complex systems and it undoubtedly has great potential for practical applications in industrial engineering.

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APPENDIX I

USER'S MANUAL

In this appendix the details of the various input and output parameters for the computer programs developed during the course of the work presented in this dissertation are given. Four programs have been developed in FORTRAN IV and run on the IBM 7044/1401 Computer System at IIT Kanpur. The first program deals with classical ABC analysis and the remaining three programs correspond to the inventory classification, and control system designed in this work. Though the latter three programs can be easily combined into a single program, due to available memory restrictions they have been retained as separate programs. The description of various FORTRAN variable names used in each program is given at the beginning of the program listing. Comment cards have been used liberally throughout the program to make it more readable.

1. Classical ABC Analysis Program

FUNCTION: This program performs the classical ABC analysis.

INPUTS: (i) The percentage of items to be put in each class has to be specified in a DATA statement in the beginning of the program. The percentage of

items in class A, B and C is denoted by APER, BPER and CPER respectively.

(ii) The data cards are designed in such a way that the same cards can be used for the first three programs. The details of the data cards are given in the next section. From each data card the name of the medicine and its annual rupee usage are read as follows:

Variable Names: NAME (I,J), USAGE (I).

Format: 40A1, F5.0

(iii) In its present form the program can handle 150 items, but this number can be increased by simply changing the DIMENSION statement. The program automatically computes the number of items for which the data has been fed. The end of the data file is indicated by the last card which has '*' punched in its 1st and 40th columns.

OUTPUT: The following information is obtained as output:

(i) The input medicine inventory data.

(ii) Results of the ABC analysis in a concise form.

(iii) Name and annual rupee usage of items belonging to each class.

2. Program for the Generation of Initial Centroids

FUNCTION : It locates initial centroids by Ball and Hall's method for given values of the weighing exponent 'EMM' and weightages 'W(J)' assigned to feature vector components, for several specified values of the inter centroidal distance 'DIST'.

INPUTS : (i) The value of the exponent 'EMM' is given in the beginning of the program in a DATA statement. Its value can be changed by simply modifying this card.

(ii) The first data card contains the total number of items, N, and the number of components in each feature vector, M.

Variable Names: N, M

Format : 2I3

(iii) The second data card contains the values of weights to be assigned to each component of the feature vector.

Variable Name : W(J)

Format : 8F5.2

(iv) The next set of data cards contains the names and feature vectors for various medicines. Corresponding to each medicine, there is one data card which is punched in the following format.

Variable Name: X(I, J)

Format : 40A1, F5.0, F3.0, 6F3.1

The first 40 columns of the data card contain the name of the medicine, the next 5 columns it's actual annual rupee usage and the subsequent 3 columns the actual expiry time of the medicine in months. The rest of the columns have the values of the remaining six feature vector components punched as either 0, 0.5 or 1.0. However, this program does not read the names of the medicines. These data cards are common for the first three programs and in each program only the relevant information is read.

(v) After the data cards for the medicines, each of the subsequent 20 data cards contains one value of the intercentroidal distance 'DIST'.

Variable Name: DIST

Format : F5.2

OUTPUT : The following information is obtained from the computer output:

(i) The input data

(ii) Inter-centroidal distances and the corresponding centroids determined for them.

(vi) The character '*' is punched on the 1st and 40th columns of the last data card. This acts as a signal to the computer that the data file has been completely read.

OUTPUT : The following information is printed out:

(i) Input data.

(ii) Total number of medicines considered for analysis.

(iii) Normalised feature vectors of all the medicines.

(iv) The total number of iterations required for convergence, final centroids, names of the items and their membership grades in each fuzzy cluster and the values of partition quality measures, H_c, $1-F_c$ and F_c.

4. Program for Combined Forecasting, Lot Sizing and Reorder Point Setting

FUNCTION : This is a common program for calculating lot sizes and reorder points for the medicines belonging to the three classes. There is no restriction on the number of medicines in input data as the program works on independent ordering strategies.

INPUT: (i) The values of K, N, ALPHA, RTRATE and SETUP are specified in the first DATA statement in the beginning of the program.

(ii) The values of T1, T2, VART1 and VART2 are specified in another DATA statement. These values will be used for'Class I' and'Class II' medicines. The following data cards are required for each medicines:

(iii) The first data card contains the following information,

Variable Name: NAME, PRICE, EXTIME, ICLASS, NCURNT

Format: 40A1, F4.2, F3.0, 2I3.

(iv) For the medicines belonging to'Class I' and 'Class II,' the rest of the Data cards are as follows:

(a) Second data card: Variable Name: T3MEAN, VART3

Format: 2F6.0

(b) Next three data cards contain monthly demand data of last three years. Each data card has the information for one year.

Variable Name: D(K1, J)

Format : 12 F6.0

(c) The last data card contains current year's demand data for 'NCURNT' months in the same format as specified in (b).

(v) The service levels for the'Class-I' and'Class II' medicines are specified by two'LOGICAL IF' statements.

(vi) For the medicines belonging to'Class III', only a single data card, after the first card specified in instruction (iii) is required:

Variable Name: ANNUAL, TTOTAL, BUFFER

Format : 3F6.0

(vii) The data cards for all the medicines are prepared as above. At a time data for a single medicine is read and processed by the program.

(viii) The end of the data file is indicated by a card whose 1st and 40th columns have the character '*' punched on them.

OUTPUT : The output contains the following information:

(i) An exception report, if the current year has ended, asking the user to update the data file.

(ii) A report is printed for every item containing its name, optimal order quantity, reorder level, unit price, cost of the lot in Rs., and number of month in which the next order is likely to be placed. The month for the next order is indicated by a number between 1 and 12.

The FORTRAN listings of these programs are given in Appendix II.

APPENDIX II

PROGRAM LISTINGS

```
 IBFTC  MAIN
C
C       ------------------------------------------------------
C
C       CLASSICAL ANNUAL RUPEE USAGE A-B-C ANALYSIS PROGRAM
C       ------------------------------------------------------
C
C
C       ------------------------------------------------------
C
C       NAME(I,J) = NAME OF THE ITEM (I)
C       USAGE(I) = ANNUAL RUPEE USAGE OF THE ITEM (I)
C       N = NUMBER OF TYPES OF ITEMS IN INVENTORY
C       NA = NUMBER OF ITEMS IN A-CLASS
C       NB = NUMBER OF ITEMS IN B-CLASS
C       NC = NUMBER OF ITEMS IN C-CLASS
C       APER = PERCENTAGE OF ITEMS BELONGINGING TO  A- CLASS
C       BPER = PERCENTAGE OF ITEMS BELONGING TO B-CLASS
C       CPER = PERCENTAGE OF ITEMS BELONGING TO C-CLASS
C
C       ------------------------------------------------------
C
C
        DIMENSION NAME(150,40),USAGE(150)
C
C       I N P U T  D A T A
C       ------------------------------------------------------
C
        DATA APER,BPER,CPER/20.,30.,50./
1001    FORMAT(/,5X,121(1H-))
        I = 1
        DATA ISTAR/1H*/
1       READ 1004,(NAME(I,J),J=1,40),USAGE(I)
1004    FORMAT(40A1,F5.0)
        IF (( NAME(I,1).EQ.ISTAR) .AND. (NAME(I,40).EQ.ISTAR)) GO TO 2
        I = I + 1
        GO TO 1
2       N = I - 1
        PRINT 1000
1000    FORMAT(/,5X,121(1H*))
        PRINT 1006
1006    FORMAT(/,42X,*THE INPUT MEDICINE INVENTORY DATA IS AS FOLLOWS*)
        PRINT 1000
        PRINT 1007
1007    FORMAT(/,25X,*ITEM NO.*,7X,*NAME OF THE ITEM*,30X,*ANNUAL RUPEE U
      1SAGE*)
        PRINT 1000
        DO 3 I = 1,N
        PRINT 1008,I,(NAME(I,J),J=1,40),USAGE(I)
1008    FORMAT(/,27X,I3,10X,40A1,8X,F9.2)
```

```
3       CONTINUE
        PRINT 1000
C
C       ARRANGING THE ITEMS IN DESCENDING ORDER OF USAGE
C       ------------------------------------------------
C
        NN = N - 1
105     ISW = 0
        DO 130 I = 1,NN
        IF (USAGE(I).GE.USAGE(I+1) )  GO TO 130
        ISW = ISW + 1
        TEMP = USAGE(I)
        USAGE(I) = USAGE(I+1)
        USAGE(I+1) = TEMP
        DO 120 J = 1,40
        TEMP = NAME(I,J)
        NAME(I,J) = NAME(I+1,J)
        NAME(I+1,J) = TEMP
120     CONTINUE
130     CONTINUE
        IF (ISW.NE.0) GO TO 105
C
C       DIVIDING THE ITEMS IN TO A,B, AND C  CLASSES
C       --------------------------------------------
C
        NA = IFIX( (FLOAT(N)*APER)/100. )
        NB = IFIX( (FLOAT(N)*BPER)/100. )
        NC = N - (NA + NB )
        ASUM = 0.0
        DO 201 I = 1,NA
        ASUM = ASUM + USAGE(I)
201     CONTINUE
        BSUM = 0.0
        DO 202 I = 1,NB
        INA = I + NA
        BSUM = BSUM + USAGE(INA)
202     CONTINUE
        CSUM = 0.0
        DO 203 I = 1,NC
        IAB = I + NA + NB
        CSUM = CSUM + USAGE(IAB)
203     CONTINUE
        TSUM = ASUM + BSUM + CSUM
        PRINT 1010
1010    FORMAT(/,40X,*THE RESULTS OF A-B-C ANALYSIS ARE AS FOLLOWS*)
        PRINT 1000
        ASPER = (ASUM*100.)/TSUM
        BSPER = (BSUM*100.)/TSUM
        CSPER = 100.00 - (ASPER + BSPER )
        PRINT 1011,APER,ASPER
```

```
      PRINT 1012,BPER,BSPER
      PRINT 1013,CPER,CSPER
1011  FORMAT(/,25X,F6.2,2X,*PERCENT ITEMS BELONGING TO A-CLASS HAVE ANN
     1UAL RUPEE USAGE OF*,2X,F6.2,2X,*PERCENT*)
1012  FORMAT(/,25X,F6.2,2X,*PERCENT ITEMS BELONGING TO B-CLASS HAVE ANN
     1UAL RUPEE USAGE OF*,2X,F6.2,2X,*PERCENT*)
1013  FORMAT(/,25X,F6.2,2X,*PERCENT ITEMS BELONGING TO C-CLASS HAVE ANN
     1UAL RUPEE USAGE OF*,2X,F6.2,2X,*PERCENT*)
      PRINT 1000
      PRINT 1014
1014  FORMAT(/,40X,*THE FOLLOWING ITEMS BELONG TO A-CLASS*)
      PRINT 1000
      PRINT 1007
      PRINT 1000
      DO 301 I=1,NA
      PRINT 1008,I,(NAME(I,J),J=1,40),USAGE(I)
301   CONTINUE
      PRINT 1001
      PRINT 1017,ASUM
1017  FORMAT(/,40X,*TOTAL ANNUAL RUPEE USAGE OF THIS CLASS OF ITEMS =*,
     1 1X,*RS.*,F10.2)
      PRINT 1000
      PRINT 1018
1018  FORMAT(/,40X,*THE FOLLOWING ITEMS BELONG TO B-CLASS*)
      PRINT 1000
      PRINT 1007
      PRINT 1000
      NA1 = NA + 1
      NANB = NA + NB
      DO 302 I = NA1,NANB
      IB = I - NA
      PRINT 1008,IB,(NAME(I,J),J=1,40),USAGE(I)
302   CONTINUE
      PRINT 1001
      PRINT 1017,BSUM
      PRINT 1000
      PRINT 1019
1019  FORMAT(/,40X,*THE FOLLOWING ITEMS BELONG TO C-CLASS*)
      PRINT 1000
      PRINT 1007
      PRINT 1000
      NANB1 = NA + NB + 1
      DO 303 I = NANB1,N
      IC = I - (NA+NB)
      PRINT 1008,IC,(NAME(I,J),J=1,40),USAGE(I)
303   CONTINUE
      PRINT 1001
      PRINT 1017,CSUM
      PRINT 1000
      PRINT 1020,TSUM
```

```
1020    FORMAT(/,25X,*TOTAL ANNUAL RUPEE JSAGE OF ALL ITEMS =*,1X,*RS.*,
     1  F10.2)
        PRINT 1000
        STOP
        END
```

```
 IBFTC MAIN
C
C       THIS PROGRAM LOCATES INITIAL CENTRIODS BY BALL AND HALL METHOD
C       ------------------------------------------------------------
C
C
C
C       N O T A T I O N S  U S E D
C       --------------------------------------------------
C
C       N = NUMBER OF TYPES OF ITEMS IN INVENTORY
C       M = NUMBER OF COMPONENTS IN ANY FEATURE VECTOR
C       X(I,J) = COMPONENT (J) OF FEATURE VECTOR OF ITEM (I)
C       X(I,1) = ANNUAL RUPEE USAGE COMPONENT
C       X(I,2) = EXPIRY TIME COMPONENT
C       X(I,3) = LIFE SAVING IMPORTANCE COMPONENT
C       X(I,4) = SUBSTITUTABILITY COMPONENT
C       X(I,5) = MARKET AVAILABILITY COMPONENT
C       X(I,6) = STOCKOUT HISTORY COMPONENT
C       X(I,7) = STORAGE SPACE COMPONENT
C       X(I,8) = STORING METHOD COMPONENT
C       V(K,J) = COMPONENT (J) OF CENTROID VECTOR OF CLASS (K)
C       W(J)  = WEIGHTAGE ASSIGNED TO COMPONENT (J) OF F.V. OF ITEM
C       EMM = WEIGHTING EXPONENT
C       DIST = SPECIFIED INTER-CENTROIDAL DISTANCE
C       ------------------------------------------------------
C
C
C
        DIMENSION X(150,12),V(60,12),D(30),W(12),VAR(12)
        DATA EMM/1.25/
        READ(5,1001) N,M
1001    FORMAT(2I3)
        WRITE(6,1001) N,M
        READ(5,999) (W(J),J=1,M)
999     FORMAT(8F5.2)
        WRITE(6,999) (W(J),J=1,M)
        READ(5,1009) ((X(I,J),J=1,M),I=1,N)
1009    FORMAT(40X,F5.0,F3.0,6F3.1)
        WRITE(6,1109) ((X(I,J),J=1,M),I=1,N)
1109    FORMAT(40X,F8.0,7F6.1)
        EPP = 1.0/(EMM-1.0)
C
C       ADJUSTMENT OF EXPIRY TIME IN FEATURE VECTORS
C       --------------------------------------------
C
        GREAT = 0.0
        DO 4 I = 1,N
        IF (GREAT.LT.X(I,2))GREAT = X(I,2)
4       CONTINUE
        DO 5 I = 1,N
```

```
      X(I,2) = ( GREAT - X(I,2) ) / GREAT
5     CONTINUE
C
C     NORMALIZATION OF FEATURE VECTOR COMPONENTS
C     ------------------------------------------
C
      DO 35 J=1,M
      IF ( J .EQ. 2 ) GO TO 35
      GREAT = 0.0
      DO 25 I=1,N
      IF (X(I,J) .GT. GREAT ) GREAT = X(I,J)
25    CONTINUE
      DO 30 I=1,N
      X(I,J) = X(I,J) / GREAT
30    CONTINUE
35    CONTINUE
C
C     THIS PORTION LOCATES INITIAL CENTROIDS
C     IT USES BALL AND HALL METHOD
C     ----------------------------------------------------
C
      CALL VARIAN(X,N,M,VAR)
      DO 300  III = 1,20
      READ(5,1021) DIST
1021  FORMAT(F5.2)
      WRITE(6,1150) DIST
1150  FORMAT(//,5X,*FOLLOWING ARE THE CENTROIDS FOR DIST =*,F8.3)
      DO 210 J=1,M
      SUM = 0.0
      DO 200 I=1,N
      SUM = SUM + X(I,J)
200   CONTINUE
      V(1,J) = SUM/FLOAT(N)
210   CONTINUE
      K = 1
      DO 265 I=1,N
      DO 240 JK=1,K
      SUM = 0.0
      DO 230 J=1,M
      SUM = SUM + W(J) * (((X(I,J)-V(JK,J))**2) / VAR(J) )
230   CONTINUE
      IF ( SUM .LE. 1.0E-06 ) GO TO 265
      D(JK) = (SUM) ** EPP
240   CONTINUE
      DO 250 JK=1,K
      IF ( D(JK) .LT. DIST ) GO TO 265
250   CONTINUE
      K = K + 1
      DO 260 J = 1,M
      V(K,J) = X(I,J)
```

```
260     CONTINUE
265     CONTINUE
        WRITE (6,1170) ((V(KJ,J),J=1,M),KJ=1,K)
1170    FORMAT(/,20X,8F8.5)
300     CONTINUE
        STOP
        END
 IBFTC  VARIAN
C
        SUBROUTINE VARIAN(X,N,M,VAR)
C
        DIMENSION X(150,12),VAR(12)
C
        DO 320 J = 1,M
        SUM1 = 0.0
        SUM2 = 0.0
        DO 310 I = 1,N
        SUM1 = SUM1 + X(I,J) ** 2
        SUM2 = SUM2 + X(I,J)
310     CONTINUE
        SUM1 = SUM1/FLOAT(N)
        SUM2 = (SUM2/FLOAT(N) ) **2
        VAR(J) = SUM1 - SUM2
320     CONTINUE
C       NORMALIZATION OF VAR(J)
        GREAT = 0.0
        DO 330 J = 1,M
        IF ( GREAT .LT. VAR(J) ) GREAT = VAR(J)
330     CONTINUE
        DO 340 J = 1,M
        VAR(J) = VAR(J) / GREAT
340     CONTINUE
        RETURN
        END
```

```
*BFTC MAIN
C
C       FUZZY   INVENTORY   CLASSIFICATION   SCHEME
C       ------------------------------------------
C
C       ------------------------------------------------------------
C       DESIGN  OF  INVENTORY  CONTROL  SYSTEM
C       THE SYSTEM IS BASED ON FUZZY SETS THEORY
C       FUZZY ISODATA IS BEING USED FOR CLUSTERING
C       THIS PROGRAM CLASSIFIES THE ITEMS FOR EFFECTIVE CONTROL
C       ------------------------------------------------------------
C
C
C       N O T A T I O N S  U S E D
C       ------------------------------------------------------------
C
C       N = NUMBER OF TYPES OF ITEMS IN INVENTORY
C       M = NUMBER OF COMPONENTS IN ANY FEATURE VECTOR
C       K = NUMBER OF CLASSES REQUIRED TO BE FORMED
C       X(I,J) = COMPONENT (J) OF FEATURE VECTOR OF ITEM (I)
C       X(I,1) = ANNUAL RUPEE USAGE COMPONENT
C       X(I,2) = EXPIRY TIME COMPONENT
C       X(I,3) = LIFE SAVING IMPORTANCE COMPONENT
C       X(I,4) = SUBSTITUTABILITY COMPONENT
C       X(I,5) = MARKET AVAILABILITY COMPONENT
C       X(I,6) = STOCKOUT HISTORY COMPONENT
C       X(I,7) = STORAGE SPACE COMPONENT
C       X(I,8) = STORING METHOD COMPONENT
C       V(K,J) = COMPONENT (J) OF CENTROID VECTOR OF CLASS (K)
C       W(J)   = WEIGHTAGE ASSIGNED TO COMPONENT (J) OF F.V. OF ITEM
C       U(I,K) = MEMBERSHIP GRADE OF ITEM(I) IN CLASS (K)
C       V1(I,J) = THE SET OF NEW CENTROIDS REPRESENTING CLUSTERS
C       NAME(I,J) = NAME OF THE ITEM (I)
C       EPSIL = CONVERGENCE CRITERION FOR PROPER CLUSTERING
C       EMM = WEIGHTING EXPONENT
C       FC = PARTITION COEFFICIENT
C       (1.-FC) = LOWER BOUND OF (FC)
C       HC = NORMALIZED ENTROPY
C       VAR(J) = VARIANCE OF THE (J)TH. F.V. COMPONENTS
C       ------------------------------------------------------------
C
C
      DIMENSION X(150,12),V(3,12),W(12),U(150,3),PT(12),GRD(3)
      DIMENSION V1(3,12),SUNUR(150,12),SUPER(12),TU(3)
      DIMENSION NAME(150,40),VAR(12)
C
C       I N P U T  D A T A
C         ----------------------------
C
      PRINT 1000
```

```
      DATA EMM/1.25/
1000  FORMAT(/,5X,121(1H*) )
      READ 1001,M,K,EPSIL
1001  FORMAT(2I4,F8.5)
      READ 1002,((V(K1,J),J=1,M),K1=1,K)
1002  FORMAT(8F9.8)
      READ 1003,(W(J),J=1,M)
1003  FORMAT(8F5.2)
C     INPUT OF FEATURE VECTORS
      I = 1
      DATA ISTAR/1H*/
1     READ 1004, (NAME(I,J),J=1,40),(X(I,K1),K1=1,M)
1004  FORMAT(40A1,F5.0,F3.0,6F3.1)
      IF((NAME(I,1) .EQ. ISTAR) .AND. (NAME(I,40) .EQ. ISTAR))GO TO 2
      I = I + 1
      GO TO 1
2     N = I - 1
      PRINT 1005,N,M,K,EPSIL
1005  FORMAT(/,7X,*NO. OF TYPES OF ITEMS IN INVENTORY =*,I4,7X,*NO. OF
     1 FEATURE VECTOR COMPONENTS =*,I4,//,7X,*NO. OF GROUPS FORMED =*,
     2 I4,25X,*CONVERGENCE CRITERION EPSIL =*,F8.5)
      PRINT 1000
      PRINT 1006
1006  FORMAT(/,42X,*THE INPUT MEDICINE INVENTORY DATA IS AS FOLLOWS*)
      PRINT 1000
      PRINT 1007
1007  FORMAT(/,5X,*ITEM NO.*,7X,*NAME OF THE ITEM*,50X,*FEATURE VECTOR
     1 *)
      PRINT 1000
      DO 3 I=1,N
      PRINT 1008,I,(NAME(I,J),J=1,40),(X(I,J),J=1,M)
1008  FORMAT(  6X,I4,10X,40A1,5X,F8.0,7F6.1)
3     CONTINUE
      PRINT 1000
C     ADJUSTMENT OF EXPIRY TIME IN THE FEATURE VECTORS
      GREAT = 0.0
      DO 4 I=1,N
      IF (GREAT .LT. X(I,2) ) GREAT = X(I,2)
4     CONTINUE
      DO 5 I=1,N
      X(I,2) = (GREAT - X(I,2) ) / GREAT
5     CONTINUE
C
C     ------------------------------------------
C     NORMALIZATION OF THE WEIGHTAGE VECTOR
C     ------------------------------------------
C
      GREAT = 0.0
      DO  10  J =1,M
      IF ( W(J) .GT. GREAT )  GREAT = W(J)
```

```
10      CONTINUE
        DO 15 J=1,M
        W(J) = W(J) / GREAT
15      CONTINUE
        PRINT 1011
1011    FORMAT(/,55X,*THE WEIGHTAGES  ARE  AS  FOLLOWS*)
        PRINT 1000
        PRINT 1012,(W(J),J=1,M)
1012    FORMAT(30X,8F8.5)
        PRINT 1000
C
C       PRINTING  THE  INITIAL  CENTROIDS
        PRINT 1013
1013    FORMAT(/,55X,*THE INITIAL CENTROIDS ARE AS FOLLOWS*)
        PRINT 1000
        PRINT 1014,((V(K1,J),J=1,M),K1=1,K)
1014    FORMAT(20X,8F12.8)
        PRINT 1000
C
C       ------------------------------------------------------
C       NORMALIZATION OF THE FEATURE VECTOR COMPONENTS
C       ------------------------------------------------------
C
        DO 35 J=1,M
        IF (J .EQ. 2) GO TO 35
        GREAT = 0.0
        DO 25 I=1,N
        IF (X(I,J) .GT. GREAT ) GREAT = X(I,J)
25      CONTINUE
        DO 30 I=1,N
        X(I,J) = X(I,J) / GREAT
30      CONTINUE
35      CONTINUE
C
C
        PRINT 1020
1020    FORMAT(/,45X,*THE FEATURE VECTORS ARE AS FOLLOWS* )
        PRINT 1000
        PRINT 1021
1021    FORMAT(/,5X,*ITEM NUMBER*,40X,*FEATURE  VECTOR  COMPONENTS*)
        PRINT 1000
        DO 50 I=1,N
        DO 40 J=1,M
        PT(J) = X(I,J)
40      CONTINUE
        PRINT 1022,I,(PT(JJ),JJ=1,M)
1022    FORMAT( 7X,I3,7X,8F12.5)
50      CONTINUE
C
C       ------------------------------------------------------
```

```
C       COMPUTING  THE  MEMBERSHIP  GRADES
C       ------------------------------------------------------
C
        CALL VARIAN(X,N,M,VAR)
        EPP = 1.0/(EMM-1.0)
        KOUNT = 0
55      CONTINUE
        KOUNT = KOUNT + 1
        DO 80 I = 1,N
        DO 62 KK=1,K
        JOT = KK
        TEST = 0.0
        DO 61 J=1,M
        TEST = ABS(X(I,J)-V(KK,J)) + TEST
61      CONTINUE
        IF(TEST.LE. 1.0E-15 ) GO TO 72
62      CONTINUE
        DO 68 KK=1,K
        GRD(KK) = 0.0
        DO 64 J=1,M
        GRD(KK) = GRD(KK) + W(J)*(((X(I,J)-V(KK,J))**2)/VAR(J))
64      CONTINUE
        GRD(KK) = (GRD(KK)) ** EPP
        IF ( GRD(KK) .LE. 1.0E-20 ) GO TO 71
68      CONTINUE
        SUM = 0.0
        DO 69 KK=1,K
        SUM = SUM + 1./GRD(KK)
69      CONTINUE
        DO 70 KK=1,K
        U(I,KK) = 1.0 / (GRD(KK)*SUM)
70      CONTINUE
        GO TO 80
71      JOT = KK
72      DO 73 KK=1,K
        U(I,KK) = 0.0
73      CONTINUE
        U(I,JOT) = 1.0
80      CONTINUE
C
C       THIS PORTION CALCULATES NEW CENTROIDS
C       ------------------------------------------------------
C
        DO 90 KK=1,K
        CDNR = 0.0
        DO 84 I=1,N
        CDNR = CDNR + (U(I,KK))**EMM
84      CONTINUE
        DO 86 I=1,N
        DO 85 J=1,M
```

```
      SUNUR(I,J) = X(I,J)*(U(I,KK)**EMM)
85    CONTINUE
86    CONTINUE
      DO 88 J=1,M
      SUPER(J) = 0.0
      DO 87 I=1,N
      SUPER(J) = SUNUR(I,J) + SUPER(J)
87    CONTINUE
88    CONTINUE
      DO 89 J = 1,M
      V1(KK,J) = SUPER(J)/CDNR
89    CONTINUE
90    CONTINUE
C
C     TEST FOR CONVERGENCE
C     ------------------------------------------------------
C
      DO 100 KK=1,K
      CDN1 = 0.0
      DO 91 J=1,M
      CDN1 = (V1(KK,J) - V(KK,J))**2 + CDN1
91    CONTINUE
      CDN = SQRT(CDN1)
      IF(CDN .GE. EPSIL) GO TO 110
100   CONTINUE
      GO TO 120
110   DO 115 KK=1,K
      DO 114 J=1,M
      V(KK,J) = V1(KK,J)
114   CONTINUE
115   CONTINUE
      GO TO 55
C
C     ------------------------------------------------------
C     INSTRUCTIONS FOR PRINTING
C     ------------------------------------------------------
C
120   PRINT 1100
1100  FORMAT(//,50X,*ITERATIONS CONVERGED - ANALYSIS SUCCESSFUL*)
      PRINT 1000
      PRINT 1120 , KCUNT
1120  FORMAT(//,28X,*TOTAL NUMBER OF ITERATIONS REQUIRED =*,I3)
      PRINT 1000
      PRINT 1121
1121  FORMAT(/,50X,*THE FINAL CENTROIDS ARE AS FOLLOWS*)
      PRINT 1000
      PRINT 1125,((V1(KK,JJ),JJ=1,M),KK=1,K)
1125  FORMAT(/,8F12.8)
      PRINT 1000
      PRINT 1130
```

```
1130    FORMAT(//,40X,*MEMBERSHIP GRADES ARE AS FOLLOWS*)
        PRINT 1000
        PRINT 1132
1132    FORMAT(//,5X,*ITEM NUMBER*,15X,*ITEM NAME*,30X,*GROUP I*,
     1  15X,*GROUP II*,15X,*GROUP III*)
        PRINT 1000
        DO 130 I =1,N
        DO 125 KK=1,K
        TU(KK) = U(I,KK)
125     CONTINUE
        PRINT 1140,I,(NAME(I,J),J=1,40),(TU(KJ),KJ=1,K)
1140    FORMAT(/,7X,I5,5X,40A1,3F21.8)
130     CONTINUE
        PRINT 1000
        CALL QUALTI(U,N,K,FC,HC)
        FCL = 1.0 - FC
        PRINT 1150,EMM,K,FC,FCL,HC
1150    FORMAT(//,10X,*WEIGHTING EXPONENT(EMM) =*,F8.4,10X,*NO. OF CLUST
     1RS(K) =*,I3,10X,*PARTITION COEFFICIENT(FC) =*,F8.5,//,10X,*LOWER
     2OUND ON (HC) = (1.-FC) =*,F8.5 ,10X,*NORMALIZED ENTROPY(HC) =*,
     3 F8.5)
        PRINT 1000
        STOP
        END
C
C
 IBFTC VARIAN
C
        SUBROUTINE VARIAN(X,N,M,VAR)
        DIMENSION X(150,12),VAR(12)
        DO 320 J = 1,M
        SUM1 = 0.0
        SUM2 = 0.0
        DO 310 I = 1,N
        SUM1 = SUM1 + X(I,J) ** 2
        SUM2 = SUM2 + X(I,J)
310     CONTINUE
        SUM1 = SUM1/FLOAT(N)
        SUM2 = (SUM2/FLOAT(N) ) **2
        VAR(J) = SUM1 - SUM2
320     CONTINUE
C       NORMALIZATION OF VAR(J)
        GREAT = 0.0
        DO 330 J = 1,M
        IF ( GREAT .LT. VAR(J) ) GREAT = VAR(J)
330     CONTINUE
        DO 340 J = 1,M
        VAR(J) = VAR(J) / GREAT
340     CONTINUE
        RETURN
```

```
      END
C
 IBFTC QUALTI
C
C
      SUBROUTINE QUALTI(U,N,K,FC,HC)
      DIMENSION U(150,3)
      FC = 0.0
      DO 5 I=1,N
      TEMP = 0.0
      DO 10 KK = 1,K
      TEMP = U(I,KK)**2 + TEMP
10    CONTINUE
      FC = FC + TEMP
5     CONTINUE
      FC = FC/FLOAT(N)
      HC = 0.0
      DO 20 KK = 1,K
      DO 20 I = 1,N
20    HC = HC + U(I,KK)*ALOG(U(I,KK))
      HC = -(HC/FLOAT(N) )
      RETURN
      END
```

```
MAIN

PROGRAM FOR FORECASTING - DYNAMIC LOTSIZING - REORDER POINT CALCULATIONS
-----------------------------------------------------------------------


-------------------------------------------------
THIS PROGRAM CALCULATES VARIOUS PARAMETERS REQUIRED FOR
MEDICINE INVENTORY CONTROL FOR ALL CLASSES OF ITEMS
-------------------------------------------------


K = NUMBER OF YEARS FOR WHICH PAST DEMAND DATA IS AVAILABLE
N = NUMBER OF PERIODS IN A YEAR
ALPHA = SMCOTHING CONSTANT
AVERGE(J) = MONTHLY AVERAGE OF (K) YEARS FOR THE PERIOD (J)
YEARAV(I) = AVERAGE MONTHLY DEMAND IN YEAR (I)
RATIO(J) = DEMAND IN LAST MOST YEAR IN PERIOD(J)/BASE VALUE OF(J)
RBAR = SMOOTHED VALUE OF THE RATIO (J) SERIES
TREND = TREND PER MONTH
BASE SERIES CONTAINS THE SEASONALITY INFORMATION
RATIO (J) SERIES INFORMS ABOUT SHIFT FROM BASE (J)
(RBAR) REPRESENTS THE SMOOTHED RATIO VALUES
TREND  GIVES THE TREND-CORRECTIONS FOR COMING YEAR
D(I,J) = DEMAND FOR PERIOD (J) OF YEAR (I)
FRCST(J) = FORECAST FOR MONTH (J)
NCURNT = NUMBER OF PERIODS OF CURRENT YEAR FOR WHICH DATA AVAILABLE
PRICE = PURCHASE PRICE OF ITEM IN RS./UNIT
SETUP = SET UP COST IN RS./ORDER
CARCST = CARRYING COST IN RS./ITEM/PERIOD
RERATE = PERCENT RATE OF RETURN ON CAPITAL FOR THE FIRM
TOTCST(J) = TOTAL COST FOR PERIODS UP TO (J)
UTCST(J) = COST PER UNIT FOR LOT COVERING REQUIREMENTS UP TO (J)
QOPT = OPTIMAL LOT SIZE FOR MINIMUM (UTCST)
CSTMIN = MINIMUM TOTAL COST PER UNIT
EXTIME = EXPIRY TIME IN MONTHS
ERROR = FORECAST VARIANCE AROUND DEMAND
ANNUAL = YEARLY DEMAND IN UNITS
ICLASS = INDICATES THE CLASS OF THE ITEM
-------------------------------------------------------------

DIMENSION D(4,12),AVERGE(12),YEARAV(3),BASE(12),RATIO(12)
DIMENSION FRCST(24),TOTCST(12),UTCST(12),NAME(40)

I N P U T   D A T A
-------------------------------------------------

DATA K,N,ALPHA,RTRATE,SETUP/3,12,0.33,12.,40./
DATA T1MEAN,T2MEAN,VART1,VART2/2.,7.,1.,2./
```

```
      DATA ISTAR/1H*/
      KOUNT = 1
      PRINT 1000
1000  FORMAT(/,2X,115(1H-))
      PRINT 1010
1010  FORMAT (/,2X,*ITEM NO.*, 1X,*NAME OF THE ITEM*,24X,*OPTIMAL*,5X,
     1 *REORDER*,6X,*UNIT*,4X,*AMOUNT IN*,4X,*NO. OF*,3X,*NEXT ORDER*,/
     2 51X,*ORDER*,7X,*LEVEL*,8X,*PRICE*,3X,*RS. TO BE*,4X,*PERIODS*,2X
     3 *TO BE PUT*,/,51X,*QUANTITY*,17X,*IN RS.*,2X,*INVESTED*,5X,*COVE
     4ED*,2X,*IN MONTH*)
      PRINT 1000
1     READ 1001,(NAME(J),J=1,40),PRICE,EXTIME,ICLASS,NCURNT
1001  FORMAT(40A1,F4.2,F3.0,2I3)
      IF((NAME(1).EQ.ISTAR).AND.(NAME(40).EQ.ISTAR)) GO TO 180
      IF ( ICLASS .EQ. 1 )  SERVCE = 0.95
      IF ( ICLASS .EQ. 2 )  SERVCE = 0.99
      IF ( ICLASS .EQ. 3 ) GO TO 110
      READ 1002, T3MEAN,VART3
1002  FORMAT(2F6.0)
      READ 1003, ((D(K1,J),J=1,N),K1=1,K)
1003  FORMAT(12F6.0)
C
C     MONTHLY AVERAGES ARE COMPUTED IN THIS PORTION
C     ---------------------------------------------
C
      DO 15 J = 1,N
      SUM = 0.0
      DO 10 I = 1,K
      SUM = SUM + D(I,J)
10    CONTINUE
      AVERGE(J) = SUM / FLOAT(K)
15    CONTINUE
C
C     COMPUTING THE TREND/MONTH
C     -------------------------
C
      DO 25 I = 1,K
      SUM = 0.0
      DO 20 J = 1,N
      SUM = SUM + D(I,J)
20    CONTINUE
      YEARAV(I) = SUM / FLOAT(N)
25    CONTINUE
      TREND = (YEARAV(K)-YEARAV(1))/FLOAT(N*(K-1))
      NUP = N/2
C     (N) MUST BE AN EVEN NUMBER
C
C     DEVELOPING THE BASE SERIES AS FOLLOWS
C     -------------------------------------
C
```

```
      DO 35 J = 1,NUP
      CJ = FLOAT(J) - 0.5
      CORECT = CJ * TREND
      N1 = NUP - J + 1
      BASE(N1) = AVERGE(N1) + CORECT
      N2 = NUP + J
      BASE(N2) = AVERGE(N2) - CORECT
35    CONTINUE
C
C     COMPUTING AND SMOOTHING RATIOS AND FORECAST ERROR
C     -------------------------------------------------
C
      RATIO(1) = D(K,1)/BASE(1)
      RBAR = RATIO(1)
      FRCST(2) = BASE(2) * RBAR
      ERROR = 0.0
      DO 50 J = 2,N
      RATIO(J) = D(K,J)/BASE(J)
      RBAR = ALPHA*RATIO(J) + (1.0-ALPHA) * RBAR
      NP = J + 1
      IF ( J.EQ.N ) NP = 1
      FRCST(J+1) = BASE(NP) * RBAR
      ERROR = ERROR + ((FRCST(J)-D(K,J))/30.0)**2
50    CONTINUE
      IF (NCURNT.EQ.12) GO TO  200
      IF (NCURNT .EQ. 0 ) GO TO 70
      KCU = K + 1
      FRCST(1) = FRCST(N+1)
      READ 1002,(D(KCU,J),J=1,NCURNT)
      DO 60 J = 1,NCURNT
      RATIO(J) = D(KCU,J)/BASE(J)
      RBAR = ALPHA*RATIO(J) + (1.0-ALPHA)*RBAR
      ERROR = ERROR + ((FRCST(J) - D(KCU,J))/30.0) **2
      FRCST(J+1) = RBAR * BASE(J+1)
60    CONTINUE
      ERROR = ERROR/FLOAT(N+NCURNT-1)
C
C     DYNAMIC LOT-SIZING BY PART PERIOD ALGORITHM (MODIFIED P.P.A.)
C     -------------------------------------------------------------
C
70    CARCST = (PRICE*RTRATE)/(100.*FLOAT(N))
      FRCST(1) = RBAR * BASE(NCURNT+1)
      QUANT = FRCST(1)
      TOTCST(1) = SETUP + CARCST*(0.5)*FRCST(1)
      UTCST(1) = TOTCST(1)/QUANT
      J = 2
75    NB = NCURNT + J
      NC = NB - N
      IF(NC.EQ.1) NB = NC
      FRCST(J) = RBAR*BASE(NB) + FLOAT(J-1)*TREND
```

```
      IIT = J-1
      TOTCST(J) = TOTCST(IIT) + (FLOAT(J)-0.5)*FRCST(J)*CARCST
      QUANT = QUANT + FRCST(J)
      UTCST(J) = TOTCST(J) / QUANT
      IF (UTCST(J).GE.UTCST(IIT)) GO TO 80
      J = J + 1
      IF (FLOAT(J).GT.EXTIME) GO TO 85
      GO TO 75
80    QOPT = QUANT - FRCST(J)
      CSTMIN = UTCST(IIT)
      GO TO 90
85    J = J - 1
      QOPT = QUANT
      CSTMIN = UTCST(J)
90    IQOPT = IFIX( QOPT + 1. )
      QOPT = FLOAT(IQOPT)
      QOPT = FLOAT(IQOPT)
      AMOUNT = PRICE * QOPT
      ICOVER = IIT
      MONTH = NB
      GO TO 98
C
C     THIS PORTION CALCULATES REORDER LEVEL
C     ------------------------------------------------------------
C
C     T1MEAN = MEAN OF THE DISTRIBUTION OF (T1)
C     T2MEAN = MEAN OF THE DISTRIBUTION OF (T2)
C     T3MEAN = MEAN OF THE DISTRIBUTION OF (T3)
C     VART1 = VARIANCE OF THE DISTRIBUTION OF (T1)
C     VART2 = VARIANCE OF THE DISTRIBUTION OF (T2)
C     VART3 = VARIANCE OF THE DISTRIBUTION OF (T3)
C     TTOTAL = TOTAL LEAD TIME IN DAYS (T1+T2+T3)
C     VARTOL = VARIANCE OF TOTAL LEAD TIME
C     SERVCE = DESIRED SERVICE LEVEL
C     DLEAD = AVERAGE DEMAND DURING THE LEAD TIME
C     BUFFER = TIME IN DAYS FOR WHICH BUFFER STOCK IS TO BE KEPT
C     REODER = REORDER LEVEL
C
C     ------------------------------------------------------------
C
110   IF ( NCURNT .EQ. 12 )  GO TO 200
      READ 1005,ANNUAL,TTOTAL,BUFFER
1005  FORMAT(3F6.0)
      Q = SQRT((2.0*100.*ANNUAL*SETUP)/(PRICE*RTRATE))
      IQ = IFIX(Q  + 1.)
      Q = FLOAT(IQ)
      TIME = (Q*FLOAT(N))/ANNUAL
      IF (TIME .GT. EXTIME ) GO TO 120
      QOPT = Q
      ICOVER = IFIX(TIME)
```

```
      GO TO 125
120   Q = (ANNUAL*EXTIME)/FLOAT(N)
      IQ = IFIX(Q + 1.)
      QOPT = FLOAT(IQ)
      ICOVER = IFIX(EXTIME)
125   NC = NCURNT + ICOVER - N
      MONTH = NCURNT + ICOVER + 1
      IF ( NC .GE. 0 ) MONTH = NC + 1
      AMOUNT = PRICE * QOPT
      REORDER = IFIX(((ANNUAL*(TTOTAL+BUFFER))/365.)+1.)
      GO TO 130
98    VARFOR = ERROR
      TTOTAL = T1MEAN + T2MEAN + T3MEAN
      VARTOL = VART1 + VART2 + VART3
C
C
      DMDRAT = (QOPT)/( 30.*FLOAT(ICOVER) )
      DLEAD = DMDRAT * TTOTAL
      VRDLED = VARFOR*(TTOTAL**2) + VARTOL*(DMDRAT**2) + VARTOL*VARFOR
C
C     REORDER LEVEL BY TCHEBYCHEFF INEQUALITY
C     ---------------------------------------------------------------
C
      CONFID = SQRT(1./(4.5*(1.-SERVCE)))
      REORDER = IFIX(DLEAD + CONFID*SQRT(VRDLED) + 1.)
130   CONTINUE
      PRINT 1020,KOUNT,(NAME(J),J=1,40),QOPT,REORDER,PRICE,AMOUNT,
     1 ICOVER,MONTH
1020  FORMAT(/,2X,I3,5X,40A1,F8.0,4X,F8.0,3X,F8.2,2X,F8.2,7X,I2,7X,I2)
      PRINT 1000
      GO TO 150
200   PRINT 2003
2003  FORMAT(/,20X,*YEAR ENDED - PLEASE UPDATE YOUR FILES*)
      GO TO 160
180   PRINT 2005
2005  FORMAT(//,35X,*END  OF  DATA  FILE*)
      GO TO 160
150   KOUNT = KOUNT + 1
      GO TO 1
160   STOP
      END
```